

Math 113

Final Exam, May 10, 2006, (2 hours).

NAME:

SECTION:

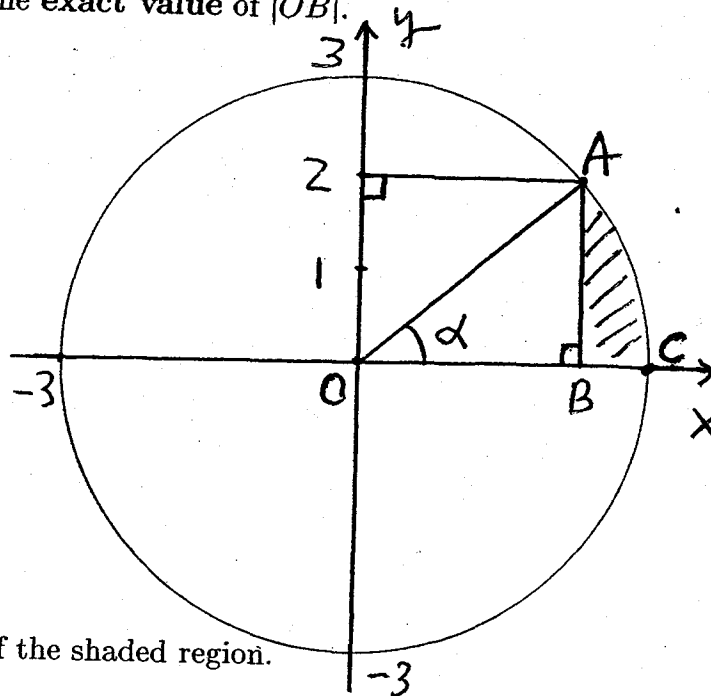
Instructor:

I	II	III	IV	V	VI	VII	VIII	IX	X	Total

TO RECEIVE CREDIT FOR AN ANSWER,

YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (NO CALCULATOR) 1) (5 points) Find the exact value of $|OB|$.



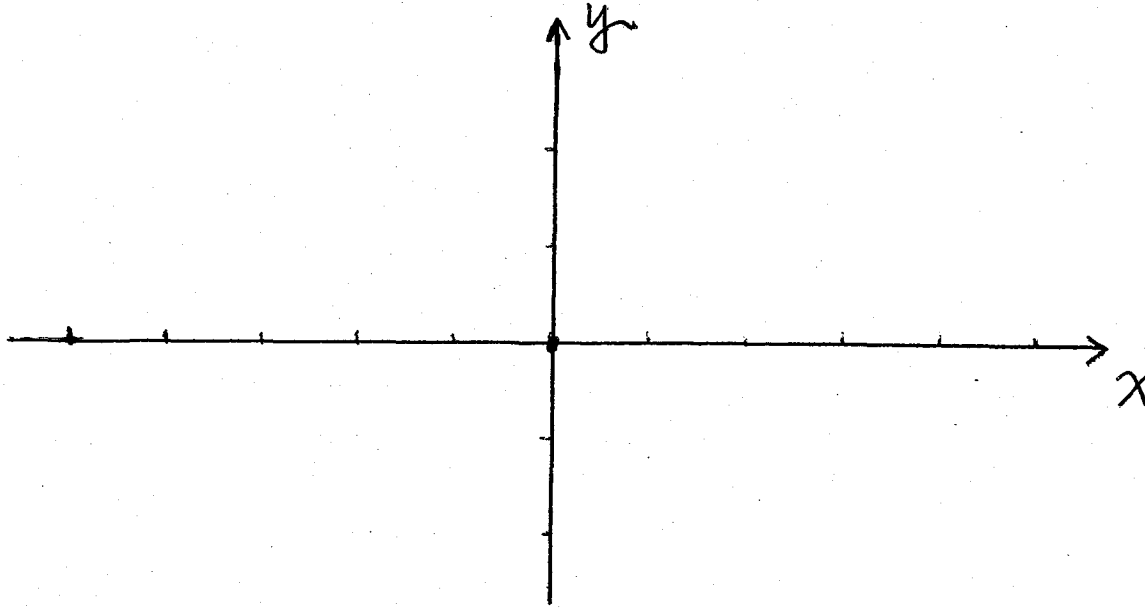
2) (10 points) Find the exact value of angle α .

3) (10 points) Find the exact value of the area of the shaded region.

4) (5 points) Find the exact value of the length of arc of circle \widehat{AC} .

II. (10 points) Find the **exact value** of $\cos(-15^\circ)$.

III. (NO CALCULATOR) 1) (15 points) Sketch the graph of $y = 2 \sin(\pi x + \frac{\pi}{4})$. Your graph must contain **at least 2 complete cycles**, and find all **exact values** of x -intercepts of your graph.



2) (15 points) Find the **exact values** of all x -intercepts of the graph of

$$y = 2 \cos\left(2x - \frac{\pi}{6}\right) - 1.$$

IV. (NO CALCULATOR) (15 points) Let

$$\cos x = \frac{1}{3}, \quad \frac{\pi}{2} < x < 2\pi.$$

Find the exact value of $\cos \frac{x}{2}$.

V. (NO CALCULATOR) (15 points) Find exact values of all solutions x to

$$\cos(2x) + 4 \sin x = 0, \quad \cos x > 0.$$

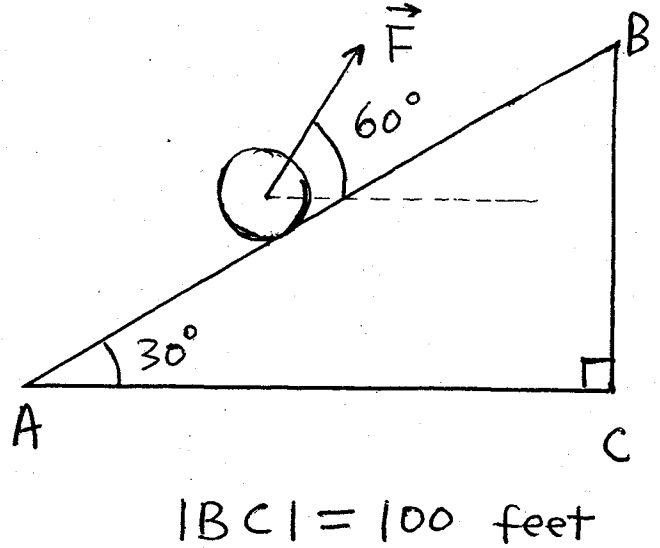
VI. (No Calculator) (20 points) You want to cross a river by a boat from west bank to east bank. The water flows 30 miles per hour from north to south. The speed of the boat relative to the water is 40 miles per hour.

Your destination is northeast (i.e. N45°E).

Find the heading of the boat.

VII. (NO CALCULATOR) A force is applied to a ball on a slope as shown below. The force \vec{F} and horizontal form an angle of 60° . The ball has weight of 30 lbs.

(1) (10 points) Find the **exact value** of minimum force \vec{F} needed to hold the ball in position. (Assume there is no surface friction.)



(2) (10 points) Find the **exact value** of work done by the force \vec{F} in (1), for pulling the ball from A to B. Assume that $|BC| = 100$ feet.

VIII. (NO CALCULATOR) 1) (6 points) Express $1 + 3i$ in the polar form.

2) (9 points) Express all roots of $z^2 = -(1 + 3i)$ in the polar form.

3) (10 points) Express $(-1 + i\sqrt{3})^{21}$ in the rectangular form.

IX. (Calculator allowed) (20 points) Recall that x° (or $0x^\circ$) stands for the direction measured as x degrees clockwise from north.

An airplane is flying at the speed of 100 miles per hour relative to the air. Assume that the destination of the airplane is at 315° (i.e. N45°W). The wind is blowing from 210° to 30° at 30 miles per hour.

Find the ground speed of the airplane.

X. (Calculator allowed) (15 points) The figure below shows 3 weighs hanging with 2 pulleys. (The system is supposed to be "ideal", weightless cables without rigidity etc.)

With the quantities as shown, find weight W_3 .

