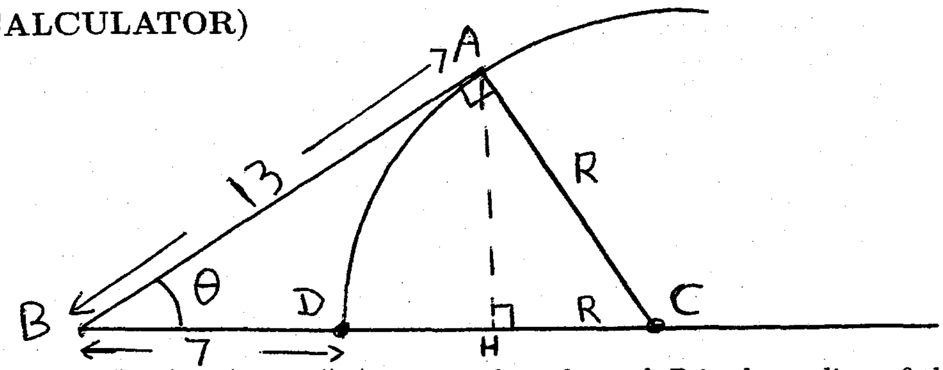
Answer sheet, same data.

Math 113 **EXAM I, Feb. 19, 2007,** (50 minutes).

NAME: SECTION: Instructor:

TO RECEIVE CREDIT FOR AN ANSWER,
YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (NO CALCULATOR) (30 points)



On the figure $|AC| = |DC| = R$ ($|\ |$ denotes length, and R is the radius of the circle).

1) Apply the Pythagorean Theorem to the right triangle BAC, and get the exact value of R.

$|BC|^2 = |AB|^2 + |AC|^2$. So $(7+R)^2 = 13^2 + R^2$

$49 + 14R + R^2 = 169 + R^2$, Therefore $R = \frac{120}{4} = \boxed{\frac{60}{7}}$

2) Give the exact values of: $\cos\theta$, $\sin\theta$ and $\tan\theta$.

$|BC| = 7 + R = 7 + \frac{60}{7} = \frac{109}{7}$

$\cos\theta = \frac{adj}{hyp} = \frac{|AB|}{|BC|} = \frac{13}{\frac{109}{7}} = \frac{91}{109}$

$\sin\theta = \frac{opp}{hyp} = \frac{|AC|}{|BC|} = \frac{\frac{60}{7}}{\frac{109}{7}} = \frac{60}{109}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{60}{91}$

3) Draw the altitude of the triangle ABC, from the vertex A to the side BC, and denote by H the point at which it meets the side BC. Give the exact values of $|AH|$ and $|BH|$.

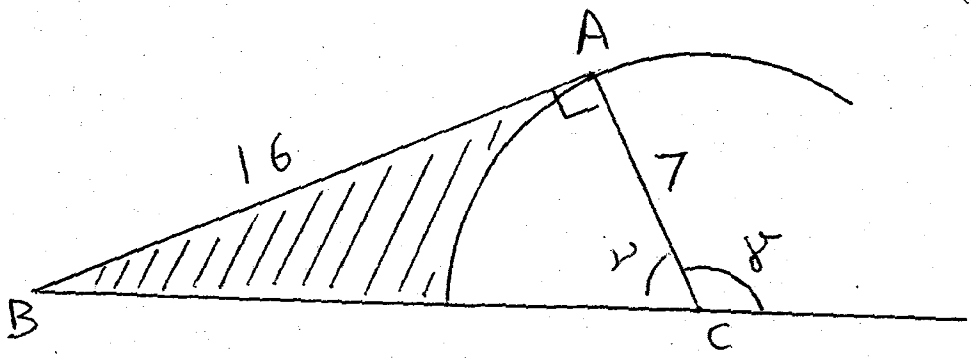
In the right triangle BHA

$|AH| = |AB| \sin\theta = \frac{780}{109}$

$|BH| = |AB| \cos\theta = \frac{1183}{109}$.

II. (With calculator).

(20 points)



The arc shown on the figure is an arc of a circle with center at C.

1) Give approximate values of the measure of the angle γ, in degrees and in radians.

ν as shown on figure. $\tan \nu = \frac{16}{7}$.

$\tan^{-1}$ gives us $\nu \simeq 66.37° \simeq 1.158$ radians

(no need to convert from degrees to radians, use $\tan^{-1}$ in degree mode and in radian mode)

$\gamma_{deg} = 180° - \nu_{deg} \simeq \boxed{113.63°}$ $\qquad$ $\gamma_{rad} = \pi - \nu_{rad} \simeq \boxed{1.983 \text{ radians}}$

2) Evaluate the area of the shaded region.

area of triangle $ABC = \frac{1}{2} |AB||AC| = \frac{1}{2}\, 16 \times 7 = 56$

area of sector $= \frac{1}{2} R^2 \nu_{rad} \simeq 28.37$ $\qquad$ $(R = 7)$.

area of shaded region $\simeq 56 - 28.37 = \boxed{27.63}$

III. What are the possible values of $\cos x$, given that $\sin x = -\frac{1}{7}$? Give the corresponding values of $\tan x$.

(15 points)

$\cos^2 x + \sin^2 x = 1$ yields $\cos^2 x + \frac{1}{49} = 1$, so

$\cos^2 x = \frac{48}{49}$. Therefore $\cos x = \pm\sqrt{\frac{48}{49}} = \pm \frac{4\sqrt{3}}{7}$, $\tan x = \frac{\sin x}{\cos x}$.

Therefore, $\boxed{\text{if } \cos x = \frac{4\sqrt{3}}{7},\ \tan x = \frac{-1}{4\sqrt{3}}.\ \text{If } \cos x = \frac{-4\sqrt{3}}{7},\ \tan x = \frac{1}{4\sqrt{3}}}$.

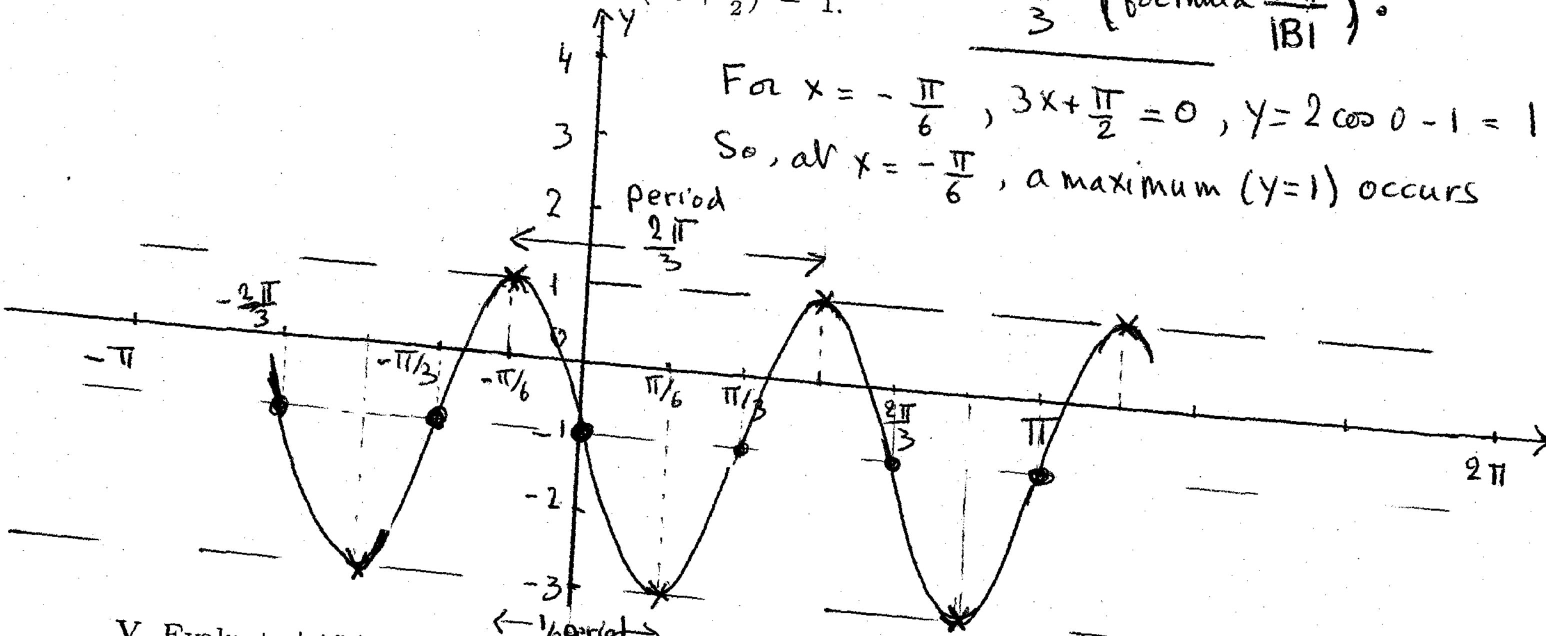
IV. Draw the graph of $y = 2\cos(3x + \frac{\pi}{2}) - 1$.

$-2-1 \le y \le 2-1$, so $-3 \le y \le 1$

period $\frac{2\pi}{3}$ (formula $\frac{2\pi}{|B|}$).

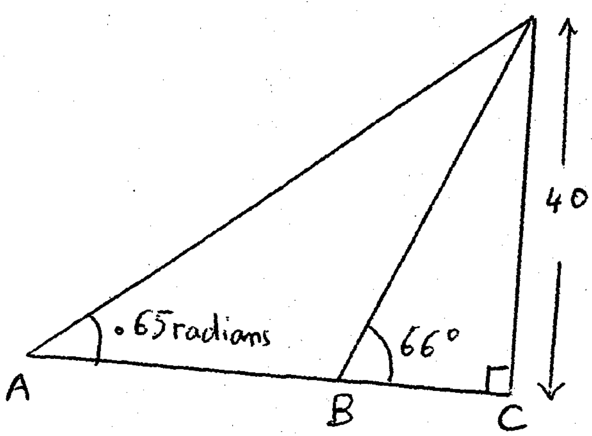
For $x = -\frac{\pi}{6}$, $3x + \frac{\pi}{2} = 0$, $y = 2\cos 0 - 1 = 1$

So, at $x = -\frac{\pi}{6}$, a maximum ($y=1$) occurs



V. Evaluate $|AB|$ (the length of the line segment AB).

(25 points)



C as shown on figure

$\tan(.65) = \frac{40}{|AC|}$

$|AC| = \frac{40}{\tan(.65)} \simeq 52.62$

(use calculator in radian mode)

$\tan 66^\circ = \frac{40}{|BC|}$, $|BC| = \frac{40}{\tan 66^\circ} \simeq 17.81$

(use calculator in degree mode)

Finally $|AB| = |AC| - |BC| \simeq 52.62 - 17.81 = \boxed{34.81}$

If you have finished the exam, then for 3 extra points: Write a 'story problem' leading to the figure above. Use back of this page.