

NAME:

SECTION: Instructor:

I	II	III	IV	V	Total
30	25	30	25	10	120

WHENEVER POSSIBLE, GIVE EXACT VALUES.
TO RECEIVE CREDIT FOR AN ANSWER,
YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (30 points) Give the EXACT values of:

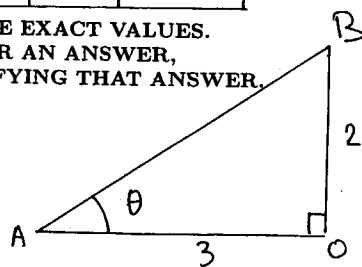
$$|AB|^2 = |AO|^2 + |OB|^2 = 13$$

the length of the line segment $AB = \sqrt{13}$

$$\sin \theta = \frac{2}{\sqrt{13}}$$

$$\cos \theta = \frac{3}{\sqrt{13}}$$

$$\tan \theta = \frac{2}{3}$$



Give an approximate value of the angle θ in radians and in degrees. Give approximate values of the length of the arc of circle from C to D , and of the area of the shaded region. O is the center of the circle.

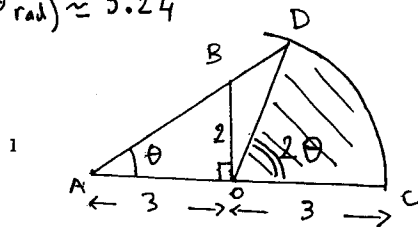
using $\tan \theta = \frac{2}{3}$: $\theta \approx 33.69^\circ \approx .59$ radians

(\tan^{-1} in degrees and radian modes)

$\angle DOC = 2\theta$ (justify it)

length of arc $\widehat{CD} = R(2\theta_{rad}) \approx 3.53$

area of shaded region = $\frac{1}{2} R^2 (2\theta_{rad}) \approx 5.24$



II. (25 points) A bike has a wheel of diameter 29". If the wheel makes 77 full rotations per minute, what is the angular speed in radians per minute? At which speed (mi/h) is the bike travelling? ($12''=1'$, $5280'=1$ mile).

for each revolution 2π radians.

$$\text{angular speed } 77 \times 2\pi \text{ rad/min} = \boxed{154\pi \text{ rad/min}}$$

$$\text{distance travelled per minute } \frac{29}{2} \times 154\pi'' = 2233\pi' \text{ (formula } R\theta_{rad}\text{)}$$

$$\text{distance travelled per hour: } \frac{2233 \times 60 \pi}{12 \times 5280} \approx 6.64 \text{ mi. } \boxed{\text{Speed} \approx 6.64 \text{ mi/h}}$$

III. (30 points) Evaluate the length h . Method imposed: Solve this problem with the tools of Chapter 2 (Right Triangles, definition of sine, cosine and tangent). Do not use tools such as the law of sines and the law of cosines to be seen in Chapter 9.

with x as shown on the figure

$$\begin{cases} h = x \tan 42^\circ & \text{(left right triangle)} \\ h = (58-x) \tan 60^\circ & \text{(right right triangle)} \end{cases}$$

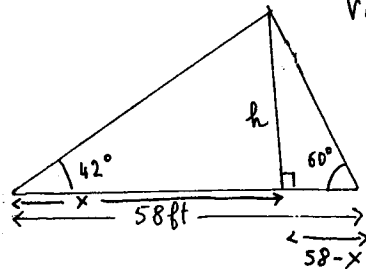
(This is not the shortest solution.)

Therefore $x \tan 42^\circ = (58-x) \tan 60^\circ$,

$$x(\tan 42^\circ + \tan 60^\circ) = 58 \tan 60^\circ,$$

$$x = \frac{58 \tan 60^\circ}{\tan 42^\circ + \tan 60^\circ}.$$

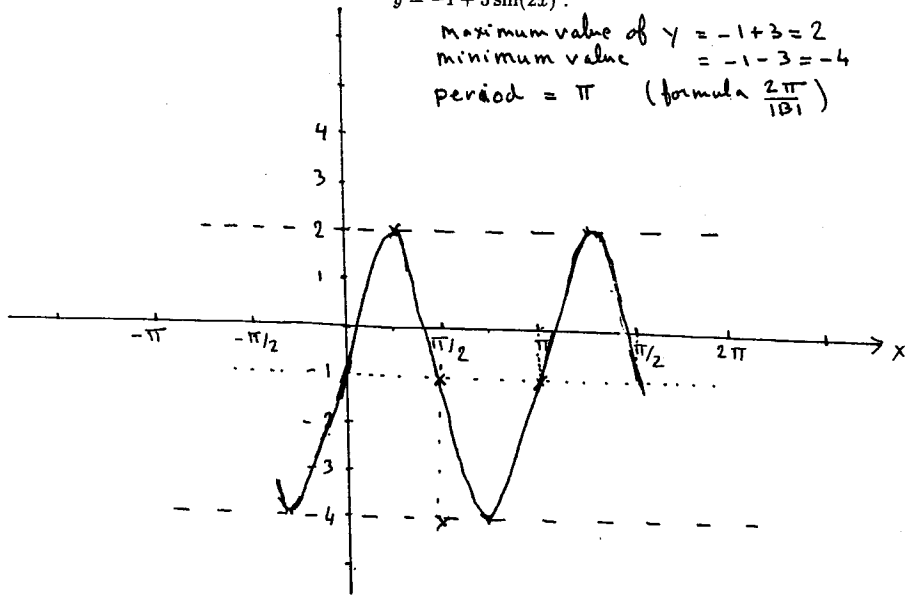
$$h = x \tan 42^\circ = \frac{58 \tan 60^\circ \tan 42^\circ}{\tan 42^\circ + \tan 60^\circ} \approx \boxed{34.36} \text{ (units)}$$



IV. (25 points) Draw the graph of

$$y = -1 + 3\sin(2x)$$

Maximum value of $y = -1 + 3 = 2$
 minimum value $= -1 - 3 = -4$
 period $= \pi$ (formula $\frac{2\pi}{|B|}$)



V. (10 points) WITHOUT CALCULATOR: Can there be an angle α such that $\tan \alpha = 3$ and $\cos \alpha = \frac{1}{3}$?

always $1 + \tan^2 = \frac{1}{\cos^2}$

But $1 + 3^2 \neq \frac{1}{(\frac{1}{3})^2} = 9$.

So there is no angle α such that $\tan \alpha = 3$, $\cos \alpha = \frac{1}{3}$.

(Note: in the above argument we do not restrict ourselves to the case $0^\circ \leq \alpha < 90^\circ$).

Alternate version

I. $|AB| = \sqrt{10}$ $\sin \theta = \frac{1}{10}$, $\cos \theta = \frac{3}{\sqrt{10}}$,

$\tan \theta = \frac{1}{3}$

$\theta \approx 18.43^\circ \approx .32$ radians
 length of the arc 1.93, area 2.90

II angular speed 146π radians/min. Speed 6.3 mi/h

III $h \approx 30.52$

IV $1 \leq y \leq -3$, period $\frac{2\pi}{3}$.