

Math 113

EXAM II, Nov. 1, 2004, (1 hour).

NAME:

SECTION:

Instructor:

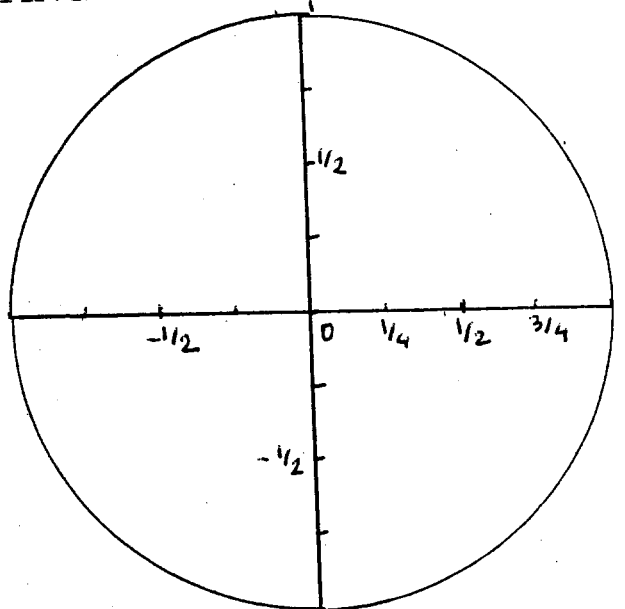
I	II	III	IV	V	Total
35	35	30	30	30	160

WHENEVER POSSIBLE, GIVE EXACT VALUES.
TO RECEIVE CREDIT FOR AN ANSWER,
YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (35 points)

1) Given $\cos^2 \theta = \frac{9}{16}$, show the possible terminal sides for the angle θ in standard position.

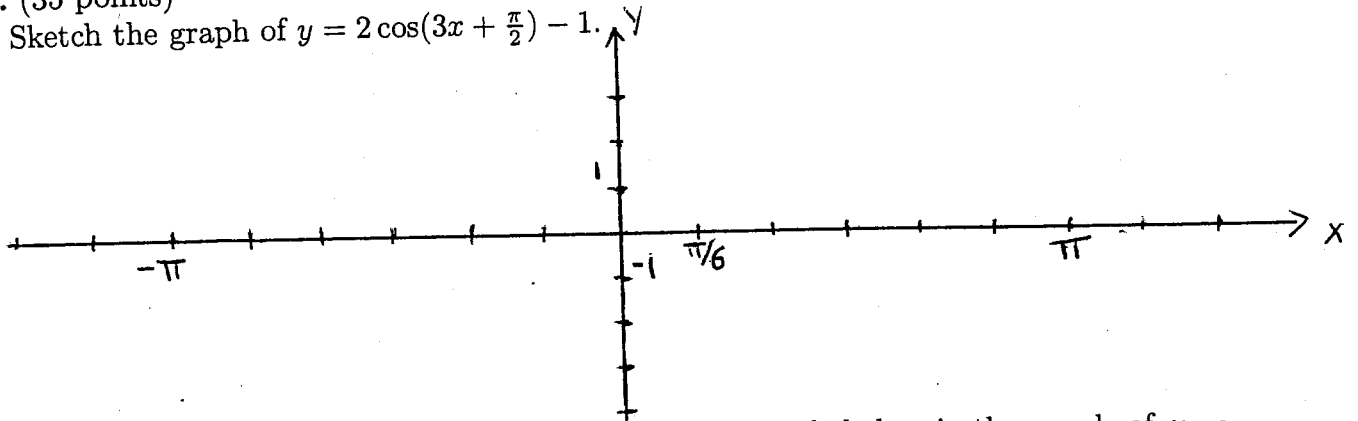
2) Let θ_0 be such that $\cos^2(\theta_0) = \frac{9}{16}$
and $\pi \leq \theta_0 \leq \pi + \frac{\pi}{2}$.
Evaluate $\sin \theta_0$, $\cos \theta_0$ and $\tan \theta_0$.



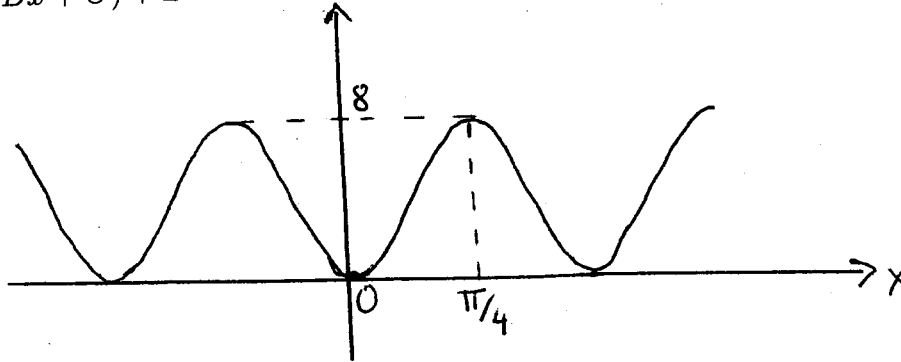
3) With θ_0 as above, is $\cos^{-1}(\cos \theta_0)$ equal to θ_0 , $-\theta_0$, $\theta_0 + 2\pi$, or $-\theta_0 + 2\pi$?

II. (35 points)

1) Sketch the graph of $y = 2 \cos(3x + \frac{\pi}{2}) - 1$.

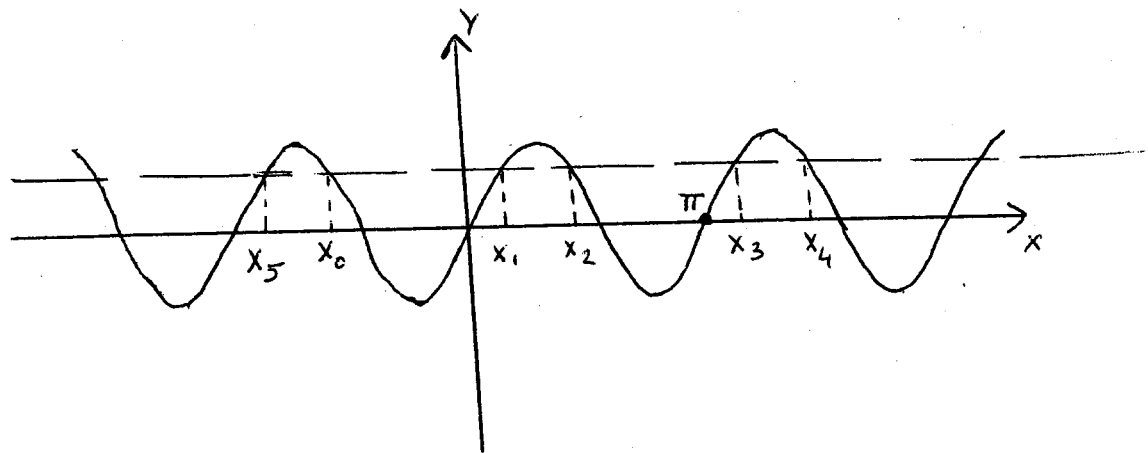


Find A, B, C, D , with A and $B > 0$, such that the graph below is the graph of $y = A \cos(Bx + C) + D$. Is the answer unique?



The graph below is the graph of $y = \sin(2x)$. What are in terms of x_0 , the values of x_1, x_2, x_3, x_4 and x_5 ?

Compare x_1 and $\sin^{-1}(\sin(2x_0))$.



III. (30 points) Set $\theta = \frac{\pi}{6} - \cos^{-1}\left(\frac{2}{3}\right)$. Evaluate exactly $\cos \theta$ and $\cos 2\theta$.

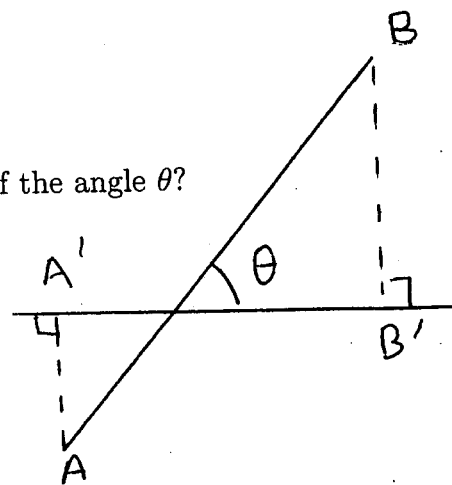
IV. (30 points) Prove the identity:

$$\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y .$$

(Even if you were not able to check the identity) Write the equality that one gets when taking $x = 20^\circ$ and $y = 10^\circ$. Make the obvious simplifications.

V. (30 points)

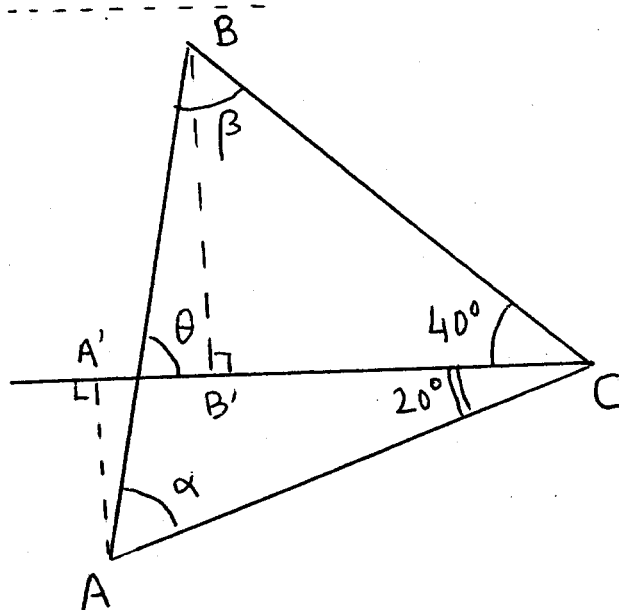
0) If $|AB| = L$, what is the length of $A'B'$, in terms of L and of the angle θ ?



On the figure: $|AC| = |BC| = L$.

1) Evaluate the angles α , β and θ .

2) Evaluate $|AB|$, and then $|A'B'|$ (using question 0).



3) Explain why also $|A'B'| = L \cos 20^\circ - L \cos 40^\circ$.

4) Using the above geometric facts and the equality found at the end of Problem IV, establish:

$$\cos 20^\circ - \cos 40^\circ = 2(\sin^2 20^\circ - \sin^2 10^\circ).$$