Math 113 **EXAM III, Dec. 1, 2004,** (1 hour).

NAME:

SECTION: Instructor:

I	II	III	IV	V	Total
24	24	24	24	24	120

WHENEVER POSSIBLE, GIVE EXACT VALUES.
TO RECEIVE CREDIT FOR AN ANSWER,
YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

24 points for each problem.

I. Set $\theta = \frac{\pi}{3} - \cos^{-1}(\frac{1}{5})$. Evaluate exactly $\sin\theta$ and $\cos(\theta + \frac{2\pi}{3})$.

Set $\alpha = \cos^{-1}\left(\frac{1}{5}\right)$. So $\cos\alpha = \frac{1}{5}$ and $0 \le \alpha \le \pi$. (See note *)

$\sin^2\alpha = 1 - \cos^2\alpha = \frac{24}{25}$. $\sin\alpha \geq 0$ since $0 \le \alpha \le \pi$, so $\sin\alpha = +\sqrt{\frac{24}{25}} = \frac{2\sqrt{6}}{5}$

$\theta = \left(\frac{\pi}{3} - \alpha\right)$, $\sin\theta = \sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta = \frac{\sqrt{3}}{2}\,\frac{1}{5} - \frac{1}{2}\,\frac{2\sqrt{6}}{5}$,

$$\boxed{\sin\theta = \frac{\sqrt{3} - 2\sqrt{6}}{10}}.$$

$$\cos\left(\theta + \frac{2\pi}{3}\right) = \cos(\pi - \alpha) = -\cos\alpha = \boxed{-\frac{1}{5}}$$

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was not asked:

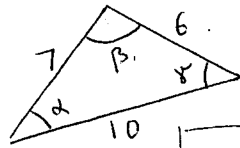
$$\sin\left(\theta + \frac{2\pi}{3}\right) = \sin(\pi - \alpha) = \sin\alpha = \frac{2\sqrt{6}}{5}$$

* $[0, \pi]$ is the range for $\cos^{-1}$, not for $\sin^{-1}$.

II. Evaluate the angles in the triangle shown on the figure.

Law of cosines: $10^2 = 7^2 + 6^2 - 2(7 \times 6)\cos\beta$

So, $\cos\beta = \frac{-15}{84}$, $\boxed{\beta \simeq 100.3^\circ}$

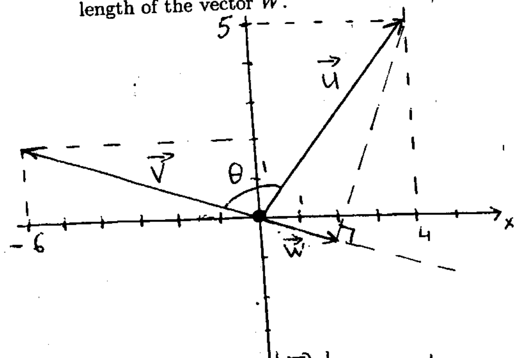


Law of sines: $\sin\alpha = \frac{6\sin\beta}{10} \simeq .5903$

α is an acute angle (obviously) so $\sin^{-1}$ gives us α

$\boxed{\alpha \simeq 36.2^\circ}$. $\gamma = 180^\circ - (100.3 + 36.2)^\circ \simeq \boxed{43.5^\circ}$

III. The vectors $\vec{U} = <4, 5>$, $\vec{V} = <-6, 2>$ and $\vec{W}$ are shown on the figure. Evaluate exactly the lengths $|\vec{U}|$ and $|\vec{V}|$ of the vectors $\vec{U}$ and $\vec{V}$. Evaluate the dot product $\vec{U} \cdot \vec{V}$. Give the exact value of the cosine of the angle θ. Evaluate exactly the length of the vector $\vec{W}$.



$|\vec{u}|^2 = 4^2 + 5^2$, $|\vec{v}|^2 = 6^2 + 2^2$

gives: $|\vec{u}| = \sqrt{41}$ $\vec{v} = \sqrt{40} = 2\sqrt{10}$

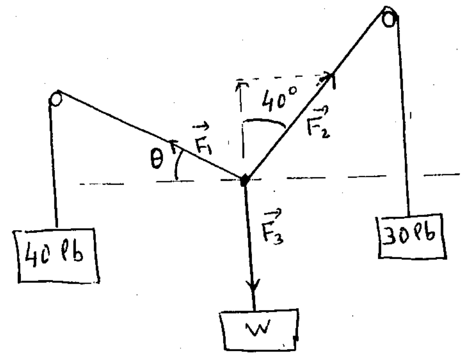
$\vec{u} \cdot \vec{v} = (4 \times (-6)) + (5 \times 2) = -14$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{-14}{2\sqrt{410}} = \frac{-7}{\sqrt{410}}$$

$$|\vec{w}| = |\vec{u}|\,|\cos\theta| = \frac{7}{\sqrt{10}}.$$

IV. Determine the weight W and the angle θ in order that the figure shows an equilibrium position.

$\vec{F_1}$, $\vec{F_2}$, $\vec{F_3}$ traction forces along the cables, as shown on figure.



$\vec{F_1} = \langle -40\cos\theta,\ 40\sin\theta \rangle$

$\vec{F_2} = \langle 30\sin 40°,\ 30\cos 40° \rangle$

$\vec{F_3} = \langle 0,\ -W \rangle$

$\vec{F_1} + \vec{F_2} + \vec{F_3} = \langle -40\cos\theta + 30\sin 40°,\ 40\sin\theta + 30\cos 40° - W \rangle = \langle 0, 0 \rangle$

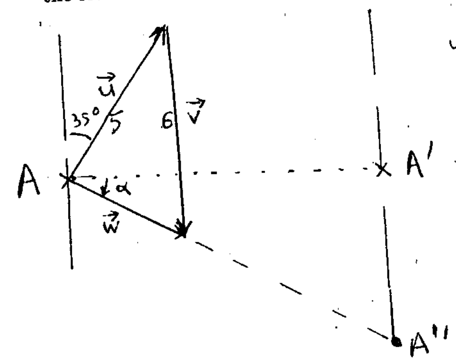
$-40\cos\theta + 30\sin 40° = 0$ yields $\cos\theta = \frac{3}{4}\sin 40° \simeq .48\ldots$ $\boxed{\theta \simeq 61°}$

$W = 40\sin\theta + 30\cos 40° \simeq \boxed{58\text{ lbs}}$

V. On a river flowing south with a current of 6 mi/h, a boat cruises at a speed of 5 mi/h. The river is .4 wide. On the western bank there is a point A. Directly across it is point A' on the eastern bank.

The boat starting from A has a compass heading of 35° (east of north). Where will the boat land? Upstream or downstream from A? At which distance? If part of the answer is obvious prior to any computation say it and explain it. How much time will crossing the river take?

Since the current is faster than the boat, the boat will obviously land south of A' (regardless of the compass heading).



On the figure
$\vec{u}$ velocity boat/river
$\vec{v}$ velocity river/ground
$\vec{w}$ velocity boat/ground.
A'' Landing point.

(on the figure different scales are used for distances (e.g. |A A''| and speed (e.g. $|\vec{w}|$))

$\vec{u} = \langle 5\sin 35°,\ 5\cos 35° \rangle$

$\vec{v} = \langle 0,\ -6 \rangle$

$\vec{w} = \vec{u} + \vec{v} = \langle 5\sin 35°,\ 5\cos 35° - 6 \rangle$

$\tan\alpha = \dfrac{5\cos 35° - 6}{5\sin 35°} \simeq -(.664)$, $\alpha \simeq -33.6°$

$|A'A''| = |AA'|\,|\tan\alpha| = .4\,|\tan\alpha| \simeq .265$

The boat will land $\boxed{.265\text{ mi south of } A'}$

crossing time $= \dfrac{\text{dist}}{\text{speed}} = \dfrac{|AA''|}{|\vec{w}|}$. One can simplify the computation by noticing that the ratio is the same as the ratio of the horizontal components. So, $\boxed{\text{crossing time} = \dfrac{.4}{5\sin 35°} \simeq .14\text{ hour}}$