Math 113 **EXAM I, Feb. 9, 2004,** (50 minutes).

NAME:

SECTION: Instructor:

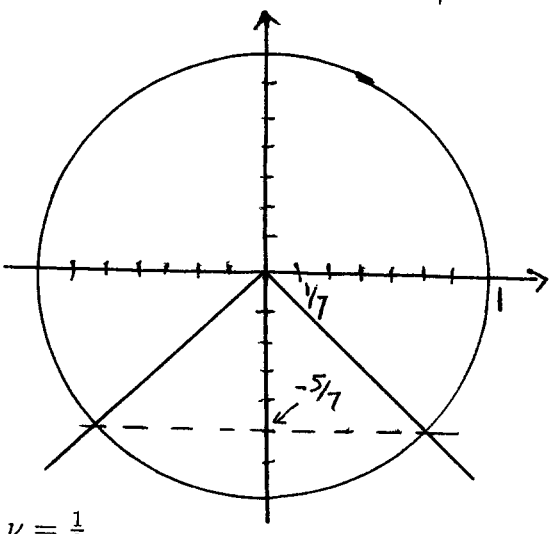
I	II	III	IV	V	TOTAL
20	10	35	25	10	100

WHENEVER POSSIBLE, GIVE EXACT VALUES.

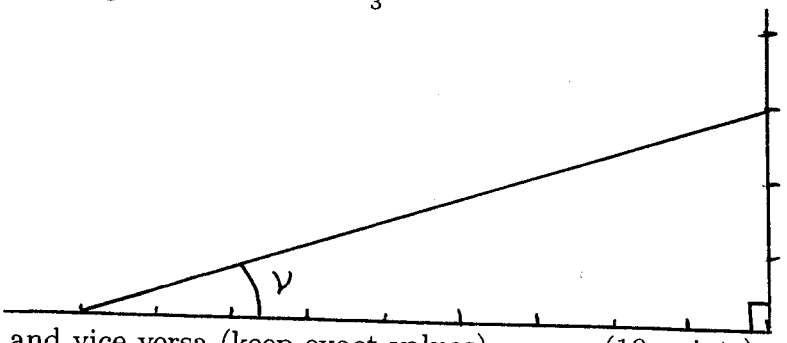
TO RECEIVE CREDIT FOR AN ANSWER, YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (20 points)

1) Draw with best possible accuracy an angle θ in standard position, such that $\sin\theta = \frac{-5}{7}$. Show all the possibilities for the terminal side.



2) Draw with best possible accuracy an angle ν with $\tan\nu = \frac{1}{3}$.



II. Convert from degrees to radians and vice versa (keep exact values). (10 points)

$12^\circ = \left(12 \times \frac{\pi}{180}\right)$ rad. $= \frac{\pi}{15}$ rad.

12 radians $= \left(12 \times \frac{180}{\pi}\right)^\circ = \left(\frac{2160}{\pi}\right)^\circ$

III. The circle whose arc is shown on the figure has a radius of length 6 (units), and O is its center. The measure of the angle $\theta = \angle ACB$ is $\frac{\pi}{8}$ radians. IN ALL THIS PROBLEM KEEP EXACT VALUES. (35 points)

1) Give the value of the angle $\angle AOB$ in degrees and in radians.

$\angle OBC = \angle OCB = \pi/8$ rad (isosceles triangle).

$\angle COB = \pi - (\pi/8 + \pi/8) = \frac{6\pi}{8}$, $\angle AOB = \pi - \angle COB = \boxed{\frac{\pi}{4}\text{rad} = 45^\circ}$

2) Evaluate the length of the line segment AB, and evaluate the area of the triangle OBC.

$|AB| = |OB| \sin\frac{\pi}{4} = 6\frac{1}{\sqrt{2}} = \boxed{3\sqrt{2}}$

area of triangle $COB = \frac{1}{2}|CO|\,|BA| = \frac{1}{2}6\times 3\sqrt{2} = \boxed{9\sqrt{2}}$ (base) (height)

3) Evaluate the length of the line segment CA, and evaluate $\tan\frac{\pi}{8}$.

$|CA| = |CO| + |OA|$, $|OA| = |AB| = 3\sqrt{2}$, so $|CA| = \boxed{6+3\sqrt{2}}$

CAB is a right triangle so: $\boxed{\tan\frac{\pi}{8} = \frac{|AB|}{|CA|} = \frac{3\sqrt{2}}{6+3\sqrt{2}} = \frac{1}{1+\sqrt{2}}}$

4) Evaluate $\cos\frac{\pi}{8}$, and $\sin\frac{\pi}{8}$ (exact values!).

$|BC|^2 = |CA|^2 + |AB|^2 = (6+3\sqrt{2})^2 + (3\sqrt{2})^2 = (36+18+36\sqrt{2}) + 18 = 36(2+\sqrt{2})$

$|BC| = 6\sqrt{2+\sqrt{2}}$.

$\cos\frac{\pi}{8} = \frac{|AC|}{|BC|} = \frac{6+3\sqrt{2}}{6\sqrt{2+\sqrt{2}}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

$\sin\frac{\pi}{8} = \frac{|AB|}{|BC|} = \frac{3\sqrt{2}}{6\sqrt{2+\sqrt{2}}} = \boxed{\frac{1}{\sqrt{4+2\sqrt{2}}}}$

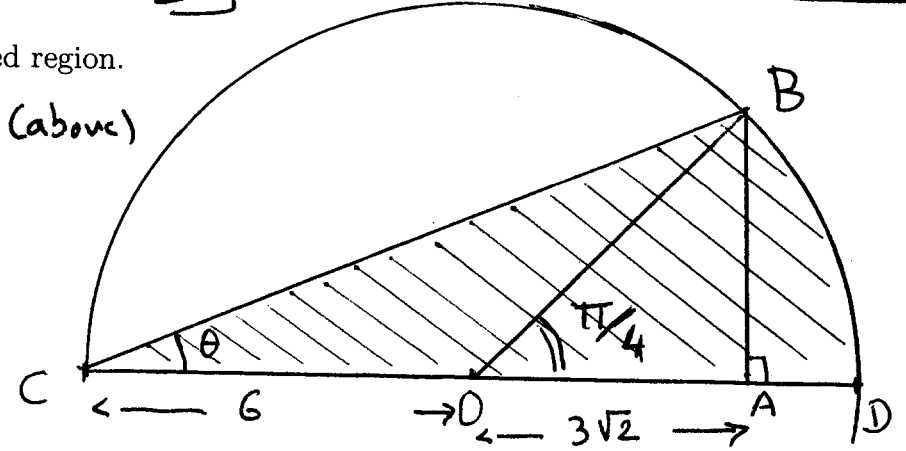
← Comments: other solution use $1+\tan^2 = \frac{1}{\cos^2}$.

simplifications were not mandatory. There are several ways to write the answers.

5) Evaluate the length of the arc of circle form D to B.

length of arc $= R\,\theta_{rad} = 6\frac{\pi}{4} = \boxed{\frac{3\pi}{2}}$

6) Evaluate the area of the shaded region.



area of triangle $OBC = 9\sqrt{2}$ (above)

area of sector OBD =

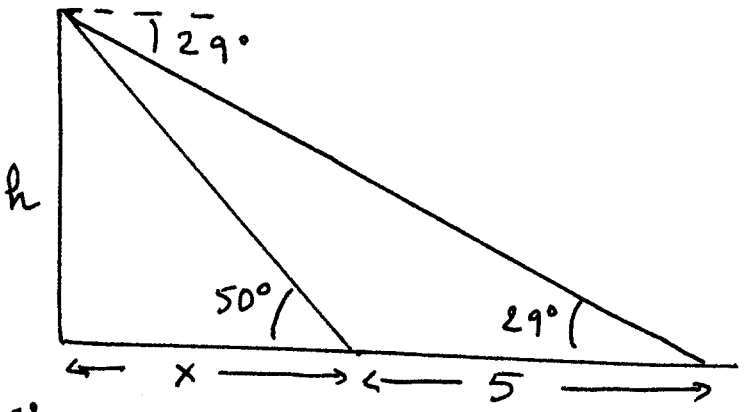
$\frac{1}{2}R^2\theta_{rad} = \frac{1}{2}36\frac{\pi}{4} = \frac{9\pi}{2}$

$\boxed{\text{area of shaded region} = 9\sqrt{2} + \frac{9\pi}{2}}$

IV. Two villages on the path of a plane are seen at an angle of depression of 50^o and 29^o. The distance between these two villages (at the same altitude) is 5 miles. At which elevation above the villages is the plane flying? (1 mile = 5280 ft) *Method imposed: Solve this problem with the tools of Chapter 2 (Right Triangles, definition of sine, cosine and tangent). Do not use tools such as the law of sines and the law of cosines to be seen in Chapter 9.*

(25 points)

with h and x as shown on figure:



$$\begin{cases} h = x \tan 50° \\ h = (x+5) \tan 29° \end{cases}$$

So $x \tan 50° = (x+5) \tan 29°$,

$x(\tan 50° - \tan 29°) = 5 \tan 29°$.

Hence $x = \dfrac{5 \tan 29°}{\tan 50° - \tan 29°}$. Since $h = x \tan 50°$, we get

$$h = \frac{5 \tan 50° \tan 29°}{\tan 50° - \tan 29°} \simeq 5.1816\ldots \text{ (mi)}$$

$h \simeq 27,359$ ft

V. If a wheel rotates with an angular velocity of 6.5 radians/sec. , how many revolutions of the wheel take place in 1 minute? If it is the wheel of a bicycle, and if the wheel has a diameter of 29″, what is the distance travelled in 1 minute?

(10 points)

1 revolution corresponds to 2π radians. So there are

$$\frac{6.5}{2\pi} \times 60 \simeq 62 \text{ rev/min}$$

distance travelled per minute = (angle of rotation in radians) $\times$ R

$= (6.5 \times 60) \times \frac{29}{2} = 5,655''$

(other solution: number of revolutions $\times$ circumference).