Math 113 **EXAM III, April 19, 2004,** (1 hour).

NAME:

SECTION: Instructor:

I	II	III	IV	V	Total
35	10	35	35	35	150

WHENEVER POSSIBLE, GIVE EXACT VALUES.

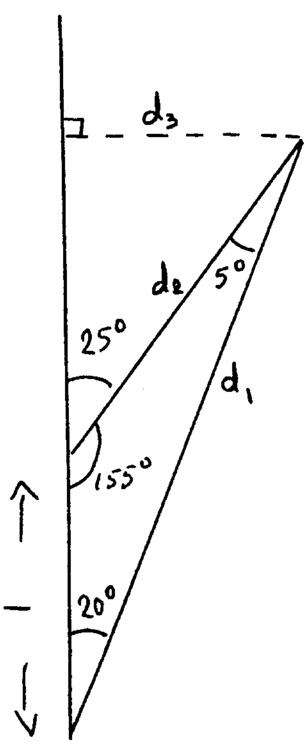
TO RECEIVE CREDIT FOR AN ANSWER,
YOU MUST SHOW WORK JUSTIFYING THAT ANSWER.

I. (35 points)

A boat is going straight north. An island is seen at an angle of 20^o east of north. After a mile, the same island is seen at an angle of 25^o east of north. How far is the boat from the island at the time of the first sighting and at the time of the second sighting?

How close to the island will the boat go, if it continues in a straight line?

(for readability, the figure is not to scale)



d_1 = distance at first sighting

d_2 = " " second sighting

d_3 = closest distance to island.

Law of sines:

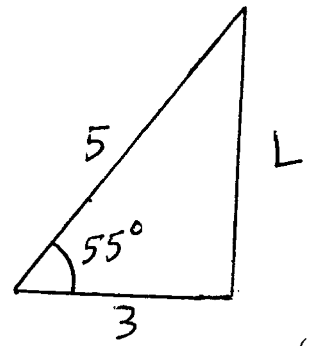
$\dfrac{d_1}{\sin 155^\circ} = \dfrac{1}{\sin 5^\circ}$ so $d_1 = \dfrac{\sin 155^\circ}{\sin 5^\circ} \simeq \boxed{4.85}$

$\dfrac{d_2}{\sin 20^\circ} = \dfrac{1}{\sin 5^\circ}$ so $d_2 = \dfrac{\sin 20^\circ}{\sin 5^\circ} \simeq \boxed{3.92}$

$d_3 = d_1 \sin 20^\circ$ (right triangle)

$d_3 \simeq \boxed{1.66}$ (mi)

II. Evaluate the length L. (10 points)



Law of cosines:

$L^2 = 3^2 + 5^2 - 2\,(3 \times 5)\cos 55°$

$L \simeq 4.1$

III. (6+29 points)

1) Find a solution to each of the following equations (keep EXACT values. You are NOT asked to find all solutions).

(i) $\sin x = \cos\frac{3\pi}{5}$. $\cos\frac{3\pi}{5} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$. So $x = \frac{\pi}{2} - \frac{3\pi}{5} = \boxed{\frac{-\pi}{10}}$ is a solution

(ii) $\sin\theta = -\cos\frac{3\pi}{5}$. $\theta = \frac{\pi}{10}$ is a solution since $\sin\frac{\pi}{10} = -\sin\left(-\frac{\pi}{10}\right)$

2) Find all solutions to the equation

$$\sin^2\theta - \cos^2\theta = \frac{1}{2},$$

in the interval $[-2\pi\ ,\ 0]$.

use $\sin^2\theta = 1 - \cos^2\theta$, and get $1 - 2\cos^2\theta = \frac{1}{2}$, so $\cos^2\theta = \frac{1}{4}$.

Therefore $\cos\theta = \pm\frac{1}{2}$.

For $\cos\theta = \frac{1}{2}$ the solutions are $\left\{\begin{array}{l} \dots, \frac{\pi}{3} - 2\pi, \frac{\pi}{3}, \frac{\pi}{3} + 2\pi, \dots \\ -\frac{\pi}{3} + 2\pi, -\frac{\pi}{3}, -\frac{\pi}{3} + 2\pi, \dots \end{array}\right.$

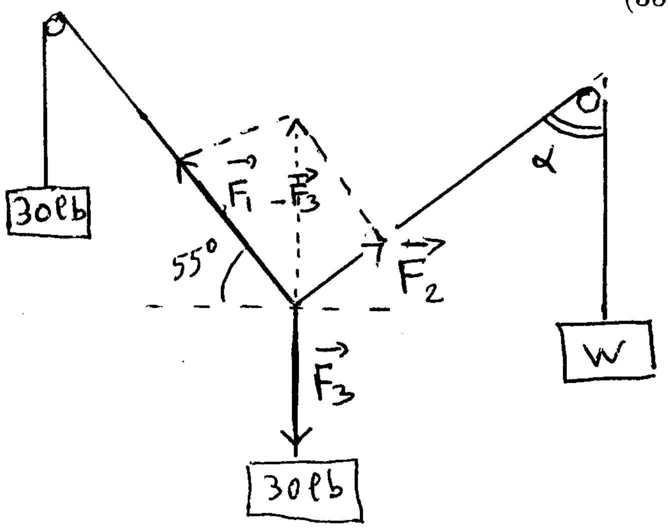
For $\cos\theta = -\frac{1}{2}$ the solutions are $\left\{\begin{array}{l} \dots, \frac{2\pi}{3} - 2\pi, \frac{2\pi}{3}, \dots \\ \dots, -\frac{2\pi}{3}, -\frac{2\pi}{3} + 2\pi, \dots \end{array}\right.$

The circled solutions are the one in the range $[-2\pi, 0]$: $\boxed{-\frac{\pi}{3},\ -\frac{2\pi}{3},\ -\frac{4\pi}{3},\ -\frac{5\pi}{3}}$

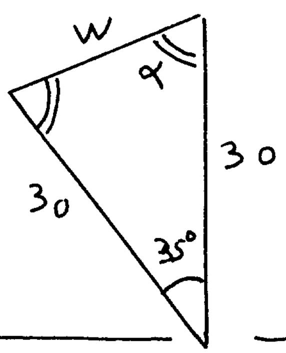
<u>other method</u> use $\sin^2\theta - \cos^2\theta = -\cos 2\theta$.

IV. Determine the weight W and the angle α so that the figure shows an equilibrium position.

(35 points)



geometric solution :



the triangle is an isoceles triangle

$2\alpha + 35° = 180°$ yields

$$\boxed{\alpha = 72.5°}$$

Law of sines $W = \dfrac{30 \sin 35°}{\sin 72.5°} \simeq \boxed{18.04}$ (lb)

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algebraic solution

$\vec{F}_1$ tension/force along left cable

$\vec{F}_2$ " " right

$\vec{F}_3$ the weight.

$\vec{F}_1 = \langle -30\cos 55°, 30 \sin 55° \rangle$

$\vec{F}_2 = \langle W \sin\alpha, W\cos\alpha \rangle$

$\vec{F}_3 = \langle 0, -30 \rangle$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$ yields

$$\begin{cases} W \sin\alpha = 30 \cos 55° \\ W\cos\alpha = 30 - 30 \sin 55° \end{cases}$$

Dividing

$$\tan\alpha = \frac{30 \cos 55°}{30 - 30 \sin 55°}$$

one gets $\alpha = 72.5°$

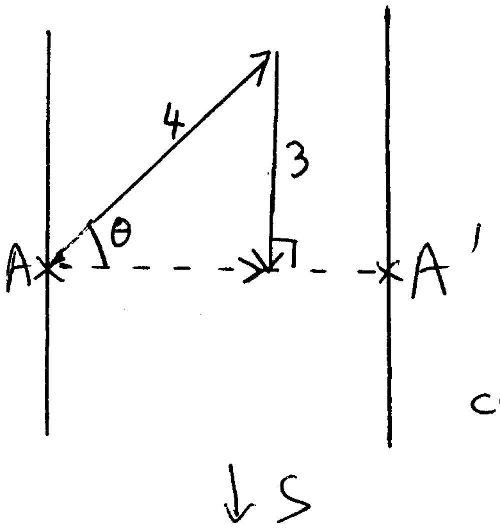
and $W = \dfrac{30 \cos 55°}{\sin 72.5°} \simeq 18.04$

V.

(25+10 points)

1) On a river flowing south with a current of 3 mi/h, a boat cruises at a speed of 4 mi/h. The river is .4 mi wide. On the ~~eastern~~ western bank of the river there is a point A. Directly across it is point A' on the ~~western~~ eastern bank.

What should be the compass heading of the boat in order to reach point A' when starting from point A. How much time will it then take to cross the river?



The figure shows the addition of the velocity vectors.

$\sin \theta = \frac{3}{4}$ gives $\theta \simeq 48.6°$

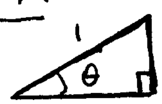
(compass heading $90° - 48.6° = 41.4°$)

ground speed $= 4 \cos \theta = 2.64$ (mi/h)

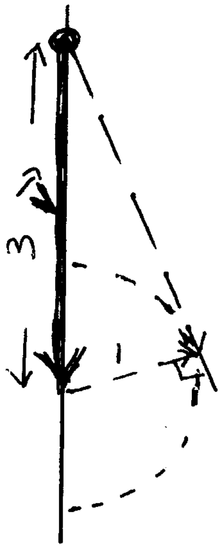
crossing time $= \frac{\text{dist.}}{\text{speed}} = \frac{.4}{2.64} \simeq .15$ hours

$\simeq 9$ min

2) Now assume that a boat (on the same river) cruises at a speed of only 1 mi/h. Starting from point A, can that boat reach point A'? Justify your answer.
Explain how the figure below is the key for answering the following question: what should the heading of the boat be in order to land as close as possible to point A'?

It is impossible to reach A' because there is no triangle such as:  (hypotenuse must be longer)

(or: even if going against the current....)



one has to add the velocity vector $\vec{V}$ shown on the left, and a vector of length 1
the tip of the resulting vector will be on the circle ....