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**Math 114 (Algebra & Trigonometry)**

Spring Semester, 2003

Exam 3: 2 May

NAME:

TA's Name:

**Be sure to show your WORK**

**Graphing & programmable calculators are NOT allowed or  
needed**

Problem	Points Received
1.(20 points)	
2.(20 points)	
3.(20 points)	
4.(20 points)	
5.(20 points)	
TOTAL (100 points)	

1. If  $u$  is between  $3\pi/2$  and  $2\pi$  and  $\cos(u) = 0.2$ , determine

(a)  $\sin(u)$  exactly:

ANSWER: \_\_\_\_\_

(b)  $\sin(u/2)$  exactly:

ANSWER: \_\_\_\_\_

(c)  $\csc(\pi/2 + u)$  exactly:

ANSWER: \_\_\_\_\_

2. Find all  $x, 0 \leq x \leq 2\pi$ , satisfying the system of equations

$$\begin{cases} \sin x - \cos x = 1 \\ \sin x + \cos x = 0 \end{cases}$$

Answers should be **exact**.

ANSWER: \_\_\_\_\_

3. Two trains from Chicago are bound for Madison and St.Louis, respectively. At a certain time, the Madison train has travelled 75 miles, and the St.Louis train has travelled 125 miles. If Chicago to Madison is 30 degrees **counterclockwise from north** and Chicago to St.Louis is 45 degrees **clockwise from south**, how far apart are the two trains at this time?

[Draw a picture!].

ANSWER: \_\_\_\_\_

4. (a) What is the domain and range of the inverse cosine function?

ANSWER: \_\_\_\_\_

- (b) What is the domain and range of the inverse tangent function?

ANSWER: \_\_\_\_\_

- (c) Sketch the graph of the inverse tangent function.

- (d) Write the complex number  $-4 - 4i$  in **polar** (trigonometric) **form** with angle argument between  $0$  and  $2\pi$  and then compute its **square roots**, that is, find all solutions of  $w^2 = -4 - 4i$  in simplest form (you may leave your answer in polar form).

ANSWER: \_\_\_\_\_

5. Verify the identity:

$$\frac{\sin 2u}{1 + \cos 2u} = \tan u.$$

### Some useful formulas

1.  $\sin^2 x + \cos^2 x = 1$

2.  $\sin(\pi/2 \pm x) = \cos(x)$

$$\cos(\pi/2 \pm x) = \mp \sin(x)$$

3.  $\sin(x \pm y) = \sin(x) \cos(y) \pm \sin(y) \cos(x)$

$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

4. Law of Cosines:  $a^2 = b^2 + c^2 - 2bc \cos A$

5. Law of Sines:  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

6.  $\sin^2(x) = (1 - \cos(2x))/2$ ,  $\sin(x/2) = \pm \sqrt{(1 - \cos(x))/2}$

$$\cos^2(x) = (1 + \cos(2x))/2, \quad \cos(x/2) = \pm \sqrt{(1 + \cos(x))/2}$$

7.  $\sin(x) \pm \sin(y) = 2 \sin(\frac{x \pm y}{2}) \cos(\frac{x \mp y}{2})$

$$\cos(x) + \cos(y) = 2 \cos(\frac{x+y}{2}) \cos(\frac{x-y}{2})$$

$$\cos(x) - \cos(y) = -2 \sin(\frac{x+y}{2}) \sin(\frac{x-y}{2})$$

8.  $\sin(x) \cos(y) = \frac{\sin(x+y) + \sin(x-y)}{2}$ ,  $\cos(x) \cos(y) = \frac{\cos(x+y) + \cos(x-y)}{2}$

$$\sin(x) \sin(y) = \frac{\cos(x-y) - \cos(x+y)}{2}$$

9.  $\sin(2x) = 2 \sin(x) \cos(x)$ ,  $\cos(2x) = \cos^2(x) - \sin^2(x)$

[End of Paper]