Caldararu

Exam 2, Math 114: Algebra and Trigonometry

November 2, 2005

No books, calculators or papers may be used, other than a hand-written note card at most $5'' \times 7''$ in size.

This examination consists of six long-answer questions. Each problem is worth sixteen points, with four points granted to everyone. Partial credits will be given only when a substantial part of a problem has been worked out. Merely displaying some formulas is not sufficient ground for receiving partial credits.

PLEASE BOX YOUR ANSWERS.

- Your name, printed:

- Your lecture section (circle one):

Nett Raghavan Fang

1	2	3	4	5	6	Total

*Name:*_____

1. Consider the functions

$$f(x) = x + 2, \quad \text{and} \quad g(x) = \frac{1}{x-1}.$$

(a) Calculate $f \circ g$, $g \circ f$, and find their domains.

(b) Find the range of $f \circ g$.

(c) Find the inverse function to $f \circ g$. Make sure to write down its domain and range.

2. (a) Show that 2 is a root of multiplicity 3 of the polynomial

$$f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8.$$

(b) Factor the polynomial $f(x)$ above completely, and find all its roots.

(c) Find a polynomial of degree 3, with real coefficients, having 0 and i (the imaginary number) as roots, and having a leading coefficient of 2.

3. Find the x-intercepts, the y-intercept, the horizontal and vertical asymptotes, and sketch a graph of the rational function

$$f(x) = \frac{x^2 - 4}{x^2 - 1}.$$

*Name:*_____

4. Solve the following equations, and show how you find your solutions:

(a) $8^{2x} = 4^{x+4}$.

(b) $\log_3(x)\log_9(3) = \log_3 \frac{x+1}{2}$. (Hint: Compute $\log_9(3)$.)

5. The speed of computers is supposed to be doubling every 2 years. If nowadays a typical computer runs at 2 GHz, how long ago was it typical to see a 4 MHz computer? (For the purpose of this problem, assume 1 GHz = 1024 MHz = 2^{10} MHz.)

6. Find all the values θ in the interval $[2\pi, 4\pi]$ for which

$$\sin\theta = -\frac{\sqrt{2}}{2}.$$