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Math 114, Lecture 1

Final Exam

December 16, 2003

YOUR NAME: \_\_\_\_\_

CIRCLE YOUR TA's NAME:           Christodouloupoulou           Lyall           Petrosyan

Do all 10 problems. Each part is worth the indicated number of points. There are 150 points altogether. Do not spend too much time on any one problem. There are 16 printed pages in this exam, including this cover page. Write neatly and show your work

Page	Possible points	Your score
Page 2	10	
Page 3	10	
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Page 16	10	
<b>TOTAL</b>	<b>150</b>	

**Problem 1** (10 points) In  $\triangle ABC$ ,  $a = \sqrt{2}$  feet,  $b = \sqrt{3}$  feet, and  $\beta = 60^\circ$ . Find the *exact* values of all possible angles  $\alpha$ .

**Problem 2** (10 points) An airplane starts at Madison and flies for 100 miles in a direction  $250^\circ$  counterclockwise from East. It then flies an additional 300 miles in a direction  $130^\circ$  counterclockwise from East. At the end of this flight, how far is the airplane from Madison?

**Problem 3** Two ropes are attached to the front of car stuck in the mud. By pulling on the ropes, the first rope exerts a force  $\vec{F}_1 = 6\hat{i} - 7\hat{j}$ , and the second rope exerts a force  $\vec{F}_2 = -3\hat{i} + 5\hat{j}$ .

(a) (5 points) What is the vector  $\vec{V}$  that gives the total force exerted by the two ropes on the car?

(b) (5 points) What is the magnitude of this total force  $\vec{V}$ ?

**Problem 3 continued:**

(c) (5 points) What is the angle between the two ropes? You can leave your answer in terms of trigonometric and inverse trigonometric functions.

(d) (5 points) What is the projection of the total force  $\vec{V}$  on the vector  $\vec{W} = 4\hat{i} - 3\hat{j}$ ?

**Problem 4** Let  $z_1 = \sqrt{3} + i$  and  $z_2 = 2 + 2i$  be two complex numbers.

(a) (5 points) Express the product  $z_1 z_2$  in the form  $a + bi$  where  $a$  and  $b$  are real numbers.

(b) (5 points) Express each of the complex numbers  $z_1$  and  $z_2$  in trigonometric form  $r \operatorname{cis}(\theta)$ .

**Problem 4 continued:**

(c) (5 points) Find the trigonometric form of the product  $z_1 z_2$ .

(d) (5 points) Use the results of parts (a) and (c) to find the exact values of  $\sin\left(\frac{5\pi}{12}\right)$  and  $\cos\left(\frac{5\pi}{12}\right)$ .

**Problem 5**

(a) (5 points) Use De Moivre's theorem to write  $(1 + \sqrt{3}i)^5$  as a complex number in the form  $a + bi$  where  $a$  and  $b$  are real numbers.

(b) (5 points) Find the three cube roots of  $-8 + 8i$ . You can leave your answer in trigonometric form.

**Problem 6** Find all the solutions for each of the following equations:

(a) (5 points)  $x^3 + x^2 + 3x - 5 = 0$ ;

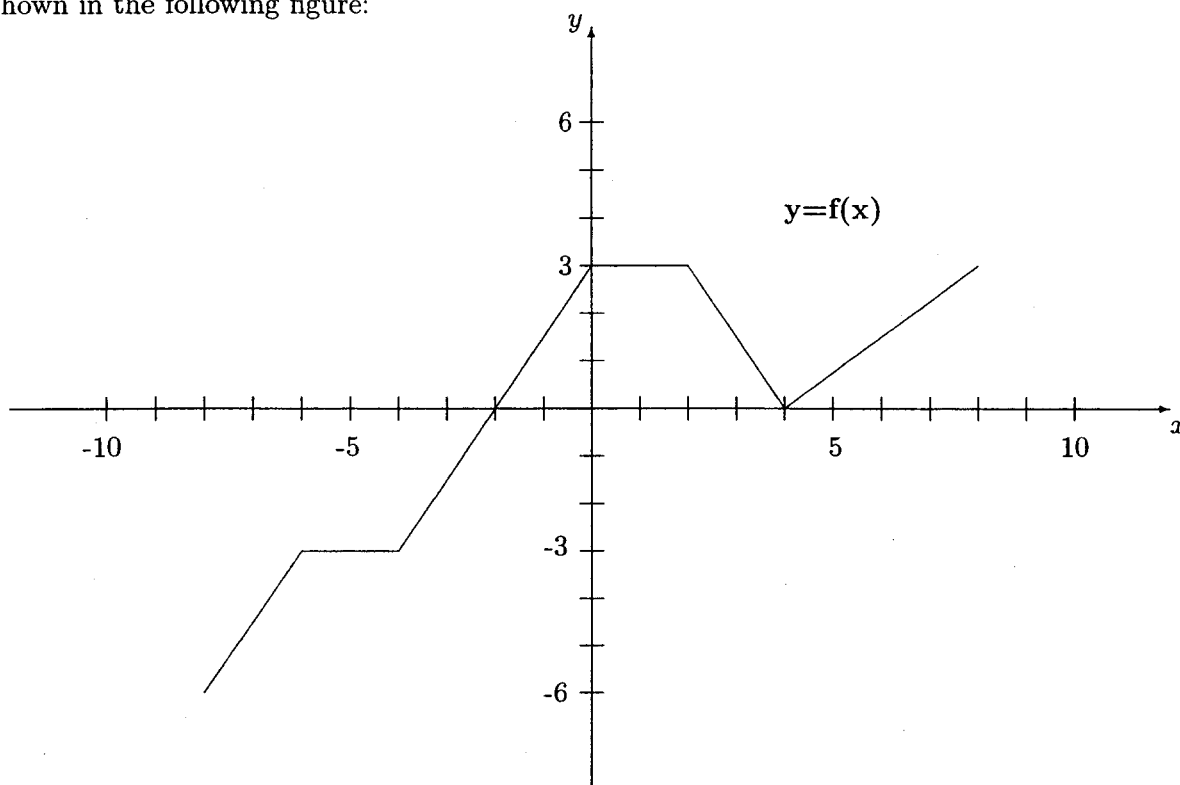
(b) (5 points)  $\log(x + 8) - \log(x + 2) = \log(x) + \log(3)$

**Problem 6 continued:**

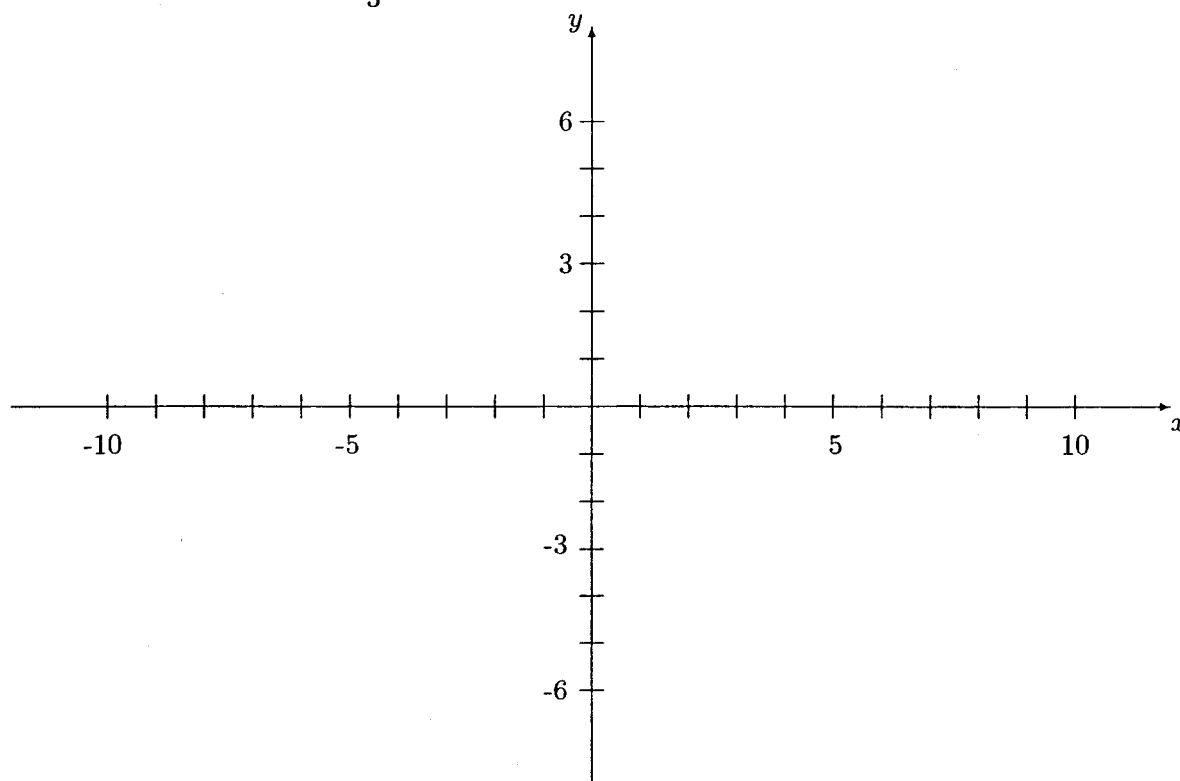
(c) (5 points)  $9^x = \left(\frac{1}{3}\right)^{x-3}$

(d) (5 points)  $2 \cos^2(x) + 9 \sin(x) - 6 = 0$

**Problem 7** (10 points) The function  $f(x)$  has domain  $-8 \leq x \leq 8$ , and the graph of  $y = f(x)$  is shown in the following figure:



Now sketch the graph of  $y = -\frac{2}{3}f(x+1)$ :

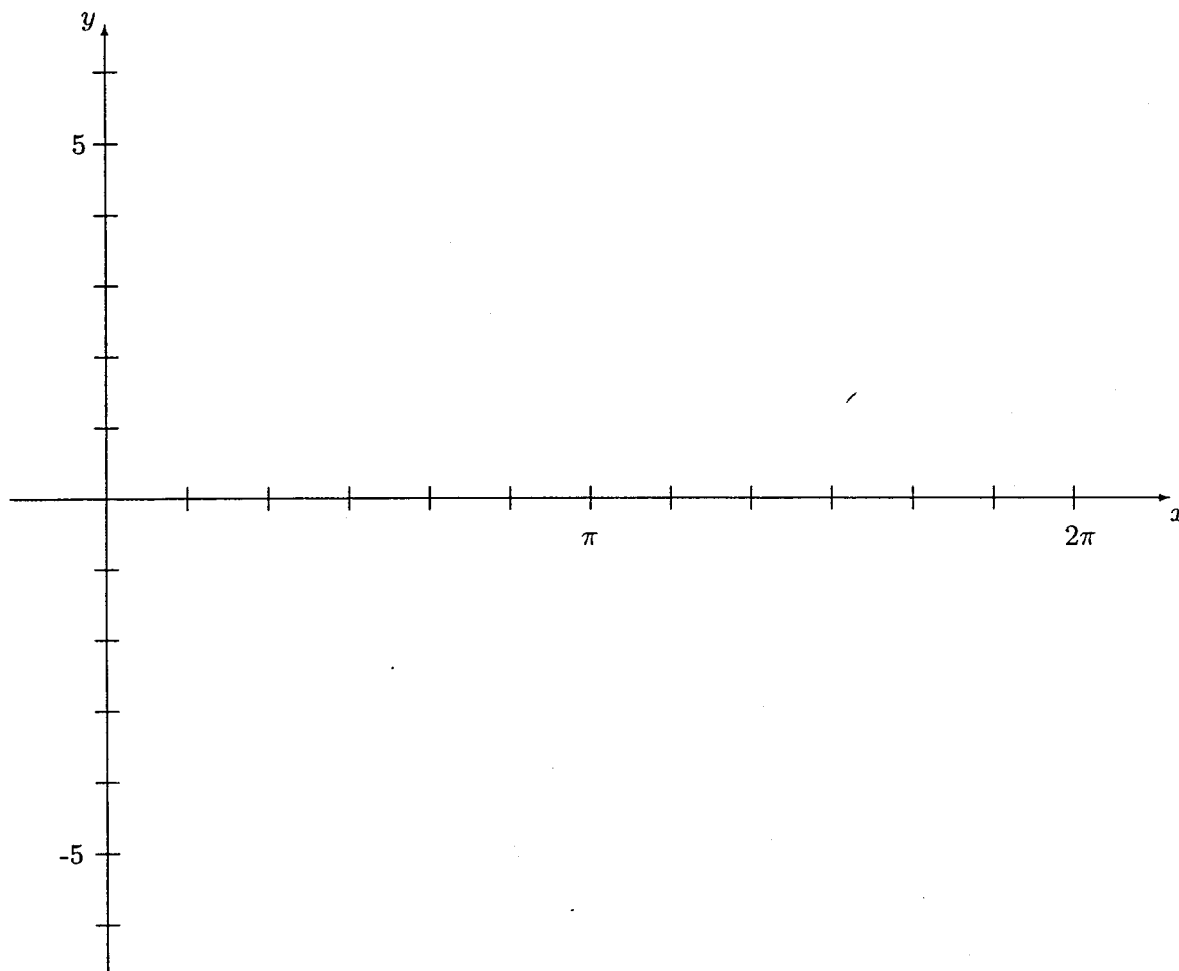


**Problem 8** Suppose that  $f(x) = 4 \sin\left(2x - \frac{\pi}{3}\right)$ .

- (a) (2 points) What is the amplitude of the function  $f(x)$ ?
- (b) (2 points) What is the period of the function  $f(x)$ ?
- (c) (2 points) What is the phase shift of the function  $f(x)$ ?
- (d) (3 points) Find all solutions of the equation  $f(x) = 0$ .
- (e) (3 points) Find all solution <sup>of</sup> ~~to~~ the equation  $f(x) = -4$ .

**Problem 8 continued:**

(f) (8 points) On the grid below, sketch the graph of  $f(x) = 4 \sin\left(2x - \frac{\pi}{3}\right)$  on the interval  $0 \leq x \leq 2\pi$ .



**Problem 9** Suppose that  $\theta$  is an angle in the second quadrant and that  $\sin(\theta) = \frac{3}{5}$ . Also suppose that  $\varphi$  is an angle in the third quadrant and that  $\tan(\varphi) = -\frac{5}{12}$ .

(a) (5 points) Find the exact values of  $\cos(\theta)$  and  $\tan(\theta)$ .

(b) (5 points) Find the exact values of  $\sin(\varphi)$  and  $\cos(\varphi)$ .

**Problem 9 continued:**

(c) (5 points) Find the exact value of  $\cos(\theta + \varphi)$ .

(d) (5 points) Find the exact value of  $\sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\varphi}{2}\right)$ .

**Problem 10** Let  $f(x) = \log_3(2x - 1) - \log_3(1 - x)$ .

(a) (3 points) What is the domain of  $f(x)$ ?

(b) (2 points) Use the properties of logarithms to write  $f(x)$  as the logarithm of a single quantity.

(c) (5 points) Find the inverse function  $f^{-1}(x)$ .