

M114 Algebra and Trigonometry
Final Exam (100pts)
Prof. Jesenko Vukadinović

Name: _____

Student ID: _____

TA's Name: _____

Section: _____

P1	
P2	
P3	
P4	
P5	
P6	
P7	
P8	
TOTAL	

Read each question carefully and try to understand it before answering the questions. You may use the back of the pages if you need extra space. **Show all the computations to get full credit. Write clearly.** Problem 1 and the problem 3 are worth 20. All other problems are worth 10 points. Good luck!!

1. (a) Verify the identity:

$$\sqrt{2} \cos(x + \pi/4) = \cos x - \sin x.$$

(b) Verify the identity:

$$\cot x - \tan x = 2 \cot 2x$$

2. Find all real solutions of the equation:

$$\sin 2t + 2 \cos t = \sin t + 1.$$

3. The number of hours of daylight $D(t)$ in Fairbank, Alaska is approximated by

$$D(t) = 6 \sin \left[\frac{2\pi}{365}(t - 79) \right] + 12,$$

with t in days, and $t = 0$ corresponding to January 1. Answer the following questions:

- (a) What is the period of the function $D(t)$? What day does $t = 370$ correspond to?
- (b) How many hours of daylight does the shortest day have? When will this occur?

(c) How many hours of daylight does the longest day have? When will this occur?

(d) How many days of the year have more than 15 hours of daylight?

4. Examine the following trigonometric function. Find the amplitude, the period, the phase shift, and then sketch the graph.

$$y = 2 \cos \left(\frac{1}{2}x + \frac{\pi}{4} \right).$$

5. Solve the following equation:

$$5^{x+\log_5 6} = 5^{x+1} + 3.$$

6. Evaluate

$$\sin \left[\cos^{-1} \left(\frac{15}{17} \right) + \tan^{-1} \left(-\frac{3}{4} \right) \right].$$

7. Express as a polynomial:

$$(\sqrt{x} - \sqrt{3})^2(\sqrt{x} + \sqrt{3})^2.$$

8. Solve the inequality

$$\frac{5x + 1}{x - 3} > 1.$$