

NAME _____

Math 130: Arithmetical Problem Solving
Midterm 2 – Monday, April 3, 2006

Instructions: You have 90 minutes for this exam. You may not use any book, notes, or calculator. If any problem seems unclear to you, ask. For computational problems, you should show your steps. For proofs, you should give enough detail to show your reasoning clearly.

1. (4 pts each)

(a) Saying that A is divisible by k means the same thing as saying that A is a _____ of k , or that k is a _____ of A .

(b) State the definition of *equation*.

(c) State the *Divisibility Lemma* (you do not have to prove it!).

2. (3 pts each)

(a) Alyssa is 14 years younger than Andrew. Agnes is nine times as old as Alyssa. Albert is 20 years younger than Agnes. If Andrew is t years old, write an algebraic expression for Albert's age.

(b) Evaluate your expression from part (a) to find Albert's age if Andrew is 23 years old.

3. (4 pts) Illustrate the identity $(a + b)^2 = a^2 + 2ab + b^2$ using a rectangular array model.

4. (5 pts each) Calculate each expression using Mental Math techniques. (Remember, your goal here is *not* to perform a pencil-and-paper computation, but rather to show on paper how you would solve the problem *entirely in your head*.)

(a) $2048 \div 128$

(b) $51^2 - 48^2$

(c) $(12340 \times 12350) - 12345^2$

5. (5 pts) Prove that the sum of two odd numbers is always even. (You may give either a picture proof or an algebraic proof.)

6. (5 pts) Determine how many zeroes are at the end of the number $500!$.

7. (10 pts) Give a Teacher's Solution, using algebra, for the following word problem:

Heather has twice as many gumdrops as Sandra. Brian has 17 more gumdrops than Sandra. The three children have a total of 93 gumdrops. How many gumdrops does Sandra have?

8. (5 pts each) Find the prime factorisation of each number. Write your answers in exponential form.

(a) 5280

(b) 7203

9. (5 pts) Simplify the expression $\frac{(m^3w)^3}{(w^2m^2)(w^0m^5)}$ as much as possible.

10. (5 pts) Prove the identity $a^m \cdot b^m = (ab)^m$.

11. (5 pts) Determine whether 397 is a prime number.

12. (8 pts) State the test for divisibility by 3. Prove it for the case of a four-digit number N .

13. (10 pts) Prove that there are infinitely many prime numbers.