

Mathematics 130  
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EXAM 1

**Problem 1** Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ ,  $A = \{1, 4, 6, 9, 11\}$ ,  $B = \{2, 3, 4, 5, 9, 10\}$ ,  $C = \{2, 6, 8, 9\}$ . Draw a Venn diagram for these sets.

**Problem 2** Use the sets in Problem 1 to find

$$(A \cup B) \cap C =$$

$$(A \cap B) \cap \bar{C} =$$

$$A \cap \bar{B} =$$

$$\bar{A} \cup \bar{C} =$$

$$A \cup \bar{B} \cup C =$$

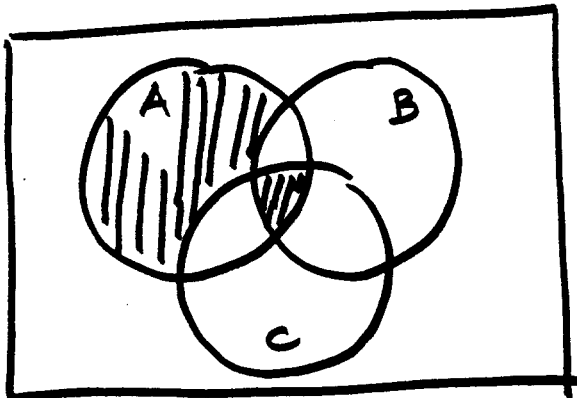
**Problem 3** (a) Find the truth table for the following implication:  $\neg(p \wedge q) \rightarrow (\neg p \wedge \neg q)$ .

(b) Show that  $(p \rightarrow q)$  and  $(\neg p \vee q)$  have the same truth tables.

**Problem 4** Use Venn diagrams to prove or disprove the statement

$$A \cup (B \cap \bar{C}) = (A \cup B) \cap \bar{C}$$

**Problem 5** (a) Describe the shaded region in the Venn diagram below.



(b) Draw a Venn diagram for  $(A \cap \overline{B \cup C}) \cup (\overline{A} \cap C)$ .

**Problem 6** Let  $U = \{1, 2, 3, \dots\}$  be the set of natural numbers. Let  $A = \{2, 4, 6, 8, \dots\}$  be the even numbers,  $B = \{b \in U \mid b = 3x + 2, 1 \leq x \leq 5\}$  and  $C = \{1, 3, 4, 6, 9, 11\}$ . Find

$$(A \cap B) \cup C =$$

$$A \cap (B \cup C) =$$

$$\overline{A} \cap B =$$

$$\overline{A} \cap C =$$

**Problem 7** Suppose  $p$  is true while  $q$  and  $r$  are false. Circle the truth value of each statement below:

T   F    $\neg p \vee \neg q$

T   F    $p \wedge (q \vee r)$

T   F    $(p \wedge r) \rightarrow q$

T   F    $(\neg p \rightarrow q) \vee r$

T   F    $\neg(p \wedge r) \rightarrow \neg q$

**Problem 8 a)** Find the truth table for  $(p \vee q) \wedge \neg p$ .

b) Let  $A$  denote the set where  $p$  is true and let  $B$  denote the set where  $q$  is true. Draw a Venn diagram and shade the region where  $(p \vee q) \wedge \neg p$  is true.

**Problem 9** Looking out in the backyard one day, I saw an assortment of chickens and pigs. Counting heads, I got 22. Counting feet, I got 68. How many chickens and how many pigs were in the yard?

**Problem 10** The soccer team, chess team, and math team are to travel on a bus together. The chess team has 12 members, the math team has 16 members, and the soccer team has 17 members. Three people are on only the math and soccer teams, two people are on only the math and chess teams, and one person is on only the chess and soccer teams. There is one person on all three teams. How many seats will the bus need for members of all three teams?