

Mathematics 130  
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EXAM 1

**Problem 1** Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ ,  $A = \{1, 4, 6, 9, 11\}$ ,  $B = \{2, 3, 4, 5, 9, 10\}$ ,  $C = \{2, 6, 8, 9\}$ . Draw a Venn diagram for these sets.

**Problem 2** Use the sets in Problem 1 to find

$$(A \cup B) \cap C =$$

$$(A \cap B) \cap \bar{C} =$$

$$A \cap \bar{B} =$$

$$\bar{A} \cup \bar{C} =$$

$$A \cup \bar{B} \cup C =$$

**Problem 3** (a) Find the truth table for the following implication:  $\neg(p \wedge q) \rightarrow (\neg p \wedge \neg q)$ .

(b) Show that  $(p \rightarrow q)$  and  $(\neg p \vee q)$  have the same truth tables.

**Problem 4** Use Venn diagrams to prove or disprove the statement

$$A \cup (B \cap \bar{C}) = (A \cup B) \cap \bar{C}$$

**Problem 5** (a) Describe the shaded region in the Venn diagram below.

(b) Draw a Venn diagram for  $(A \cap \overline{B \cup C}) \cup (\overline{A} \cap C)$ .

**Problem 6** Let  $U = \{1, 2, 3, \dots\}$  be the set of natural numbers. Let  $A = \{2, 4, 6, 8, \dots\}$  be the even numbers,  $B = \{b \in U \mid b = 3x + 2, 1 \leq x \leq 5\}$  and  $C = \{1, 3, 4, 6, 9, 11\}$ . Find

$$(A \cap B) \cup C =$$

$$A \cap (B \cup C) =$$

$$\overline{A} \cap B =$$

$$\overline{A} \cap C =$$

$$B \cup C$$

**Problem 7** Suppose  $p$  is true while  $q$  and  $r$  are false. Circle the truth value of each statement below:

T F  $\neg p \vee \neg q$

T F  $p \wedge (q \vee r)$

T F  $(p \wedge r) \rightarrow q$

T F  $(\neg p \rightarrow q) \vee r$

T F  $\neg(p \wedge r) \rightarrow \neg q$

**Problem 8 a)** Find the truth table for  $(p \vee q) \wedge (\neg p)$ .

b) Let  $A$  denote the set where  $p$  is true and let  $B$  denote the set where  $q$  is true. Draw a Venn diagram and shade the region where  $(p \vee q) \wedge \neg p$  is true.

**Problem 9** a) The ratio of marbles in a red box to that in a blue box is 4:3. There are six more marbles in the red box. We add two marbles to the blue box. What is the new ratio?

b) There is an assortment of chickens and pigs in the backyard. There are 22 heads and 68 feet. How many chickens and how many pigs are in the yard?

**Problem 10** A class has 42 students. Some of the students are on one or more teams. There is one student on all three teams. In addition, three students are on the math and soccer teams only, two students are on the math and chess teams only, and one student is on the chess and soccer teams only. The chess team has 12 members, the math team has 16 members, and the soccer team has 17 members. How many students are not on any team?