

Math 130

March 12, 2003

## First Exam

Name: \_\_\_\_\_

Section: \_\_\_\_\_

There are 100 possible points on this exam. Be sure to read each question carefully and answer the question asked. Show your work neatly and clearly—answers without justification may reduce your score. Partial credit may be given for a correct approach even if you don't get to the right answer.

**GOOD LUCK!**

Problem	Max	Score
1	17	
2	13	
3	15	
4	10	
5	13	
6	10	
7	12	
8	10	
TOTAL	100	

# Scratch Paper

1. Let  $S = \mathbb{N}$ , and consider the operation defined by

$$a \oplus b = a^2 + b^2$$

(a) Is this operation closed? Explain.

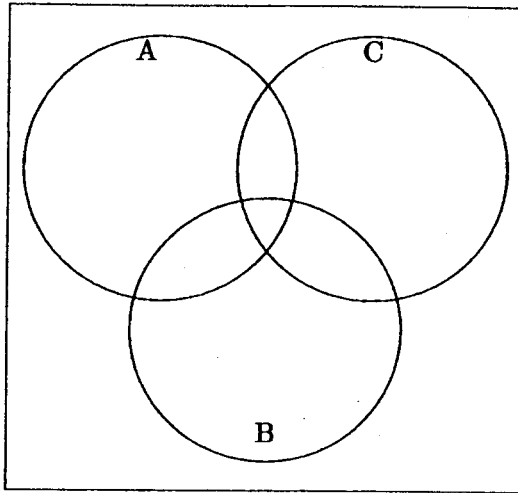
(b) Is this operation commutative? Explain.

(c) Is this operation associative? Explain.

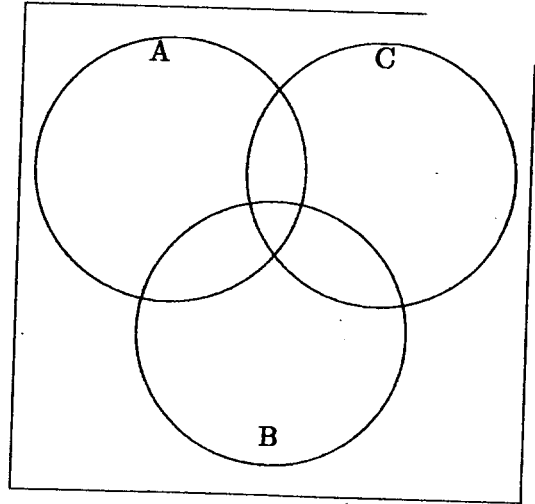
(d) Does this operation possess an identity? Explain.

2. In the following problems, shade the Venn diagram in correspondence with the set given. Be precise!

a)  $A \cap (\overline{B \cup C})$



b)  $\overline{A} \cup (B \cap \overline{C})$



3. Perform the following operations in the base indicated. Be sure to justify your computation.

(a)  $2149_{12} + 1388_{12}$

(b)  $1738_9 - 864_9$

4. Convert the number  $1843_{11}$  into its base 8 equivalent.

5. Consider the operation, defined on  $\mathbb{Q}$ , given by

$$a * b = a + b - ab$$

(a) Does this operation have an identity? If so, what is it?

(b) Is this operation invertible? If so, can you find a formula for the inverse?

6. Suppose that  $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $A = \{0, 2, 4, 6, 9\}$ , and  $B = \{1, 2, 3, 6, 7, 9\}$ . Determine the following sets.

(a)  $A \cap (\bar{A} \cup B)$

(b)  $\bar{A} \cap (A \cap B)$

7. Taylor is twelve years older than Karyn. In two years, Taylor will be twice as old as Karyn was three years ago. How old are Taylor and Karyn?

8. (This one is tough!) Suppose  $x \in \mathbb{R}$  is any real number. Show that  $0x = 0$  (i.e. show that zero times any number is always zero).  
(Hint: Use the fact that zero is the additive identity for the real numbers, and use the standard properties of addition and multiplication. Good luck!)

