

**Final Exam**

Name: \_\_\_\_\_

There are 150 possible points on this exam. Be sure to read each question carefully and answer the question asked. Show your work neatly and clearly—answers without justification may reduce your score. Partial credit may be given for a correct approach even if you don't get to the right answer. You may use calculators, compasses, and straightedges. No notes, textbooks, or other materials are allowed.

Problem	Max	Score
1a	20	
1b	15	
2	15	
3	15	
4	12	
5	12	
6	10	
7	12	
8	12	
9	15	
10	12	
11	0	
<b>TOTAL</b>	<b>150</b>	

1. A Greek Adventure

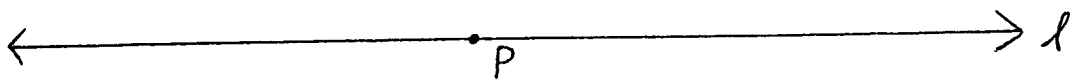
You wake up one morning to find yourself surrounded by ancient Greeks. (No, you are not at a class reunion on Langdon Street.) You are asked for your passport, which you don't have, and are promptly arrested. At your trial, you are forced to demonstrate your knowledge of Greek culture by doing the following two tasks. (Remember this is your only chance to save yourself from the wretched dungeon of geometric illiteracy!

(a) (20 points) State and prove the Pythagorean Theorem. Make sure your presentation is orderly and clear to the skeptical observers.

(b) (15 points) The prosecutor says "If the sides of a triangle  $T$  have lengths  $a$ ,  $b$ , and  $c$  satisfying  $a^2 + b^2 = c^2$ , then  $T$  is a right triangle." Then he demands "What do you think about that?!"

If he is telling the truth, give a proof. If it is possible that he is lying, give a counterexample to his claim.

2. (15 points) Given the point  $P$  on the line  $\ell$ , construct a line perpendicular to  $\ell$  and passing through  $P$ . Justify your construction.



3. (15 points) How many symmetries does a square prism have? How many of these symmetries can be expressed without using any reflections? (That is, how many symmetries are obtainable using just rotations?) Explain how you know your answer is correct.

4. (12 points) What are the 4 types of rigid motions of the plane? (No justification required.)

(i) \_\_\_\_\_

(ii) \_\_\_\_\_

(iii) \_\_\_\_\_

(iv) \_\_\_\_\_

5. (12 points) The following describe rigid motions of the plane. Assume that neither (a) nor (c) is the "do nothing" symmetry. After each one, indicate which type(s) of rigid motion it could be. No justification is required. (Just write the appropriate type or types from your list in Problem 4.)

(a) Composition of 3 rotations \_\_\_\_\_

(b) Composition of 3 reflections \_\_\_\_\_

(c) A rotation followed by a translation \_\_\_\_\_

6. (10 points) Given 2 distinct points on a sphere, how many lines (in the sense of spherical geometry) pass through the two points? Sketch the possibilities.

7. (12 points) Is there a regular polyhedron whose faces are octagons? If such a polyhedron exists, give its name. If not, give an argument for why such a polyhedron is impossible.

8. (12 points) Write a short essay on non-Euclidean geometry in the space provided. Be sure to address the following issues:

- What is non-Euclidean geometry?
- What is its early history?
- Why is it important?

9. (15 points) Suppose a (not necessarily regular) polyhedron  $P$  has  $2000x^2$  vertices,  $200x$  faces, and 398 edges. List the possible number or numbers of vertices for  $P$ .

10. (12 points) A cube  $C$  is inscribed in a sphere  $S$  of radius  $r$ . Find (exactly) the ratio

$$\frac{\text{volume of } S}{\text{volume of } C}.$$

You may use the fact that the volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ .  
(Your answer should be a number.)

11. (10 points Extra Credit) Inscribe a circle in the given triangle below.  
Justify your construction.

