

Math 132

May 15, 2003

Final Exam

Name: _____

Section: _____

Be sure to read each question carefully and answer the question asked. Show your work neatly and clearly—answers without justification may reduce your score. Partial credit may be given for a correct approach even if you don't get to the right answer.

1. Consider the following game: you flip a (not necessarily fair) coin, and if you get tails, you win \$3, but if you get heads, you lose \$2. What should the probability of getting tails be if you want your average winnings to be 0?

2. My age now is twice what your age was two years ago. At the same time, I am only 10 years older than you. What are our ages?

3. Our friend Farmer Todd wants to put up a fence to keep his cows from running off. He has 200 yards of fencing, and wants to enclose a rectangular region whose length is 30 yards less than twice the width. What are the dimensions of the region enclosed by the fence?

4. Suppose your chances of winning at blackjack are $\frac{1}{10}$. If you've played nine times and lost every time, what are your chances of winning on the tenth hand? Why? (Be sure to explain your answer!)

5. Between 1980 and 1990, there were a total of 80 shark attacks in Florida. However, between 1990 and 2000, there were over 180. Therefore, you are over twice as likely to be bitten now than 20 years ago.

Discuss how this statistic could be misleading.

6. In rolling two fair six-sided dice, what is the probability of having a two showing if the sum is odd? What about the probability of a two showing if the product is odd? Would your answer change to the second question if the dice are unfair? (Do not try to calculate the probability for the unfair case!)

7. Suppose you have given a homework assignment to your students about conditional probability. One of your students writes "to find the probability of outcome A happening given that B happens, I just count the number of ways that A and B happen, and divide by the number of ways that B can happen. So, for example, looking at rolling two six-sided dice, to figure out the probability of an even sum given that a two shows, I just count how many ways that a two shows (there are 11: (2,1) and (1,2), (2,2), (2,3) and (3,2), (2,4) and (4,2), (5,2) and (2,5), and (2,6) and (6,2)), and how many of those have an even sum (there are 5). Thus, the probability of an even sum given that a two shows is $\frac{5}{11}$." Explain the mistake in the student's reasoning.

8. Suppose we have two unfair six-sided dice, both of which have probability distributions

outcome	probability
1	$\frac{7}{16}$
2	$\frac{1}{8}$
3	$\frac{1}{8}$
4	$\frac{1}{8}$
5	$\frac{1}{8}$
6	$\frac{1}{16}$

Find the expected value for rolling these dice and adding the outcomes. ($8 * 16 = 128$)

9. Consider the following experiment. You have a bag with 10 marbles, 8 of which are white, and 2 of which are red. You remove two marbles at random, one after the other, without replacement. (So, after removing one marble, and before removing the second, there are nine marbles in the bag.) Are the two outcomes (the first marble is white) and (the second marble is red) independent or dependent? Why?

What if we put the first marble back after recording its color? Are the two outcomes independent or dependent? Why? (You do not need to compute any probabilities to answer these questions!)

10. Suppose that on the day you were born, your grandparents put \$25 in a bank account that earns 5% interest in the year. Every year after that, your grandparents put \$25 into the account on your birthday, and it earns 5%. How much money do you have on your 25th birthday?

(Hint: On your first birthday, you will have 25 (because it is your birthday) plus the 25 from the previous year, and the 5% interest it earned. So, altogether, you will have $25 + (25 + (.05)(25))$, which we can rewrite as $25 + ((25)(1 + .05)) = 25 + 25(1.05)$ or just $25 + 25(1.05)^1$. On your second birthday, you will have 25 (since it is your birthday), plus the amount from the year before plus the interest on the amount from the year before. In other words: you will have

$$25 + ((25 + 25(1.05)) + (.05)(25 + 25(1.05))),$$

which we can rewrite as (factoring $(25 + 25(1.05))$ out in the second parantheses)

$$25 + (25 + 25(1.05))(1 + .05) = 25 + 25(1.05)^1 + 25(1.05)^2)$$

