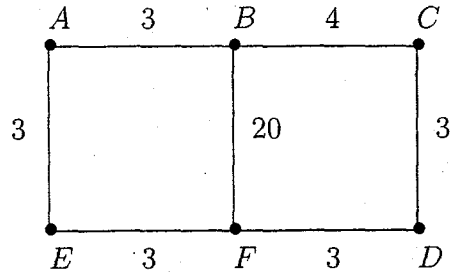


Math 141 Second Midterm Exam Thursday December 14 2006

Name _____

I	15 Points	
II	35 Points	
III	25 Points	
IV	25 Points	
Total	100 Points	

I (15 points.) Find the cheapest route in the graph below starting at vertex A , finishing at vertex A , and traversing each edge at least once. The cost of a route is computed by summing the numbers along the edges used.



II (35 points.) The following table shows the mileage between four cities: Springfield (S); Urbana (U); Effingham (E); and Indianapolis (I).

	E	I	S	U
E	-	147	92	79
I	147	-	190	119
S	92	190	-	88
U	79	119	88	-

(i) Represent this information by drawing a weighted complete graph on four vertices.

(ii) One of the Hamiltonian Circuits in this graph is *ESIUE*. Find the others. Warning: *ESIUE* is the same circuit as *SIUES*.

(Problem II continued.)

(iii) Find the cost of each the distinct Hamiltonian circuits. Which Hamiltonian circuit gives the minimum cost?

(iv) Which circuit is obtained from the nearest neighbor algorithm starting at E ? at I ? at S ? at U ?

(v) Which circuit is obtained from the sorted-edges algorithm?

(vi) Find the minimal spanning tree.

III (25 points.) Henry and Lisa play the following game. Henry chooses a row and Lisa chooses a column from the matrix

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

(Each is ignorant of the other's choice.) Then Lisa gives Henry the entry in that row and column.

(i) Find the optimal mixed strategy for Henry.

(ii) Find the optimal mixed strategy for Lisa.

(iii) What is the value of the game, i.e. how much should Henry pay Lisa before each play of the game so that in the long run either is equally likely to be ahead?

IV (25 points.) Our text explained four methods of apportionment: Hamilton's method, Jefferson's method, Webster's method, and Hill-Huntington's method.

(i) Which (if any) of these methods can lead to the Alabama paradox?

(ii) Which (if any) of these methods always satisfies the quota condition?

(iii) Three states A, B, and C have populations 729, 337, 534, respectively. Apportion a house of size 16 among them according to each of the four methods. Put your answers in the table. Below the table I have done enough arithmetic so that you don't need a calculator to solve the problem.

State	Population	Hamilton	Jefferson	Webster	Hill-Huntington
A	729				
B	337				
C	534				
	1600	16	16	16	16

$$729/100 = 7.290$$

$$729/97.1 = 7.508$$

$$729/97.45 = 7.481$$

$$729/91.0 = 8.011$$

$$337/100 = 3.370$$

$$337/97.1 = 3.471$$

$$337/97.45 = 3.458$$

$$337/91.0 = 3.703$$

$$534/100 = 5.340$$

$$534/97.1 = 5.499$$

$$534/97.45 = 5.480$$

$$534/91.0 = 5.868$$

$$\sqrt{3 \cdot 4} = \sqrt{12} = 3.464$$

$$\sqrt{4 \cdot 5} = \sqrt{20} = 4.472$$

$$\sqrt{5 \cdot 6} = \sqrt{30} = 5.477$$

$$\sqrt{6 \cdot 7} = \sqrt{42} = 6.481$$

$$\sqrt{7 \cdot 8} = \sqrt{56} = 7.483$$

$$\sqrt{8 \cdot 9} = \sqrt{72} = 8.485$$