

## Exam 2

Name: \_\_\_\_\_

TA's Name \_\_\_\_\_

Discussion Section Time \_\_\_\_\_

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

This exam contains 10 pages, and 7 problems. Before you begin, please make sure all the pages are here.

No calculators, notes, or books are allowed. The last page contains some geometric formulas that you may or may not need. If you decide to remove that page, please do so carefully.

**You must show all your work, and explain your reasoning to receive credit for your answers.**

Be sure to check your answers whenever possible.

When you are finished, please return your exam to your TA.

*Good luck!*

1. Evaluate the limit or explain why it does not exist.

(a) [5 points]  $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

(b) [5 points]  $\lim_{x \rightarrow -\infty} \frac{t}{t-5}$

1. (continued)

(c) [5 points]  $\lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{x^2 + 3}}$

(d) [5 points]  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x)$

2. [10 points] Find the values of  $a$  and  $b$  so that the following function is continuous everywhere.

$$\begin{cases} x + 1, & \text{if } x < 1 \\ ax + b, & \text{if } 1 \leq x < 2 \\ 3x, & \text{if } x \geq 2 \end{cases}$$

3. (a) [5 points] Use the definition of the derivative to find  $f'(4)$ , where  $f(x) = \frac{1}{x-1}$ .

(b) [5 points] Use your answer to part a to find an equation for the tangent to  $y = f(x)$  at  $x = 4$ .

(c) [10 points] Use the definition of the derivative to find  $g'(x)$ , where  $g(x) = \frac{x}{x-5}$ .

4. Find the indicated derivative.

(a) [5 points]  $\frac{d}{dx}[3x(x^3 - 1)]$

(b) [5 points]  $f''(x)$ , where  $f(x) = \frac{(x+1)^2}{x-1}$ .

(c) [5 points]  $\frac{dy}{dx}$ , where  $y$  is implicitly defined by  $x^3y + y^3x = 30$ .

5. [12 points] Sketch the graph of  $y = \frac{x^2 - 9}{x^3 - x}$ , labeling all intercepts and asymptotes.

6. [10 points] An object thrown vertically downward from the top of a cliff with initial velocity  $v_0$  feet per second falls approximately  $s = v_0t + 16t^2$  feet in  $t$  seconds. If it strikes the ocean below in 3 seconds with a velocity of 140 ft/s, how high is the cliff?

7. [13 points] Water is pouring into a conical tank at a rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?

## Formulas You May or May Not Need

### Geometric Formulas

*volume of a cone:*  $V = \frac{1}{3}\pi r^2 h$

*surface area of a cone:*  $S = \pi r \sqrt{r^2 + h^2}$

*volume of a sphere:*  $V = \frac{4}{3}\pi r^3$

*surface area of a sphere:*  $S = 4\pi r^2$

*volume of a right circular cylinder:*

$$V = \pi r^2 h$$

*surface area of a right circular cylinder:*

$$S = 2\pi r h$$