

Math 171 – Calculus with Algebra and Trigonometry. I

Fall 2006 – Midterm 2

Name: _____

TA: _____

VERSION A

- (a) The exam has 6 problems.
- (b) Show your work for every problem. Answers without justification will not receive credit.
- (c) Calculators are not allowed.
- (d) There is a blank page at the end of the exam for if you need it.

Problem	Points	Score
1	30	
2	30	
3	30	
4	20	
5	20	
6	20	
Total	150	

1. (30 points) Differentiate the following functions.

(a)

$$f(x) = 3x^3 + 5x^2 - 12x + 1.$$

(b)

$$g(x) = \frac{\sqrt{x} + x}{\sqrt{x} - x}$$

(c)

$$h(x) = \left(x + \frac{1}{x}\right)^{16}$$

2. Consider the equation

$$x^3 + 3xy + y^3 = 1.$$

(a) (10 points) Show that the point $(\frac{1}{2}, \frac{1}{2})$ lies on the graph of this equation.

(b) (20 points) Compute $\frac{dy}{dx}$ at this point.

3. (30 points) A local artist is making a statement by blowing up a balloon that has the shape of a cube. After several hours the balloon contains 64 cubic feet of air (so the side-length is 4 feet), and the well trained artist blows in air at a rate of $\frac{1}{5}$ cubic feet per minute. What is the rate at which the side-length of the balloon increases?

4. (20 points) Compute the following limits.

(a)

$$\lim_{x \rightarrow 3} \frac{x^2 - 3x}{2x - 6}$$

(b)

$$\lim_{t \rightarrow 2^+} \frac{|t| + 2}{|t| - 2}$$

5. (20 points) Draw a picture of a circle that explains why

$$\lim_{t \rightarrow 0} \frac{\sin(t)}{t} = 1.$$

In other words, show that for small values of t the sine of t is very close to t . (you do not need to write sentences, just draw a picture and label $\sin(t)$ and t).

6. (20 points) Suppose that $f(x)$ is a differentiable function, and its derivative satisfies $f'(x) > 2$ for all real numbers x . If it is given that $f(1) = 1$, what can we say about $f(5)$? Explain your answer.

