

Math 171 – Calculus with Algebra and Trigonometry. I

Fall 2006 – Final Exam

Name: \_\_\_\_\_

TA: \_\_\_\_\_

VERSION A

- (a) The exam has 6 problems.
- (b) Show your work for every problem. Answers without justification will not receive credit.
- (c) Calculators are not allowed.
- (d) There is a blank page at the end of the exam for if you need it.

Problem	Points	Score
1	30	
2	30	
3	40	
4	40	
5	40	
6	20	
Total	200	

1. Compute the following limits.

(a) (10 points)

$$\lim_{x \rightarrow 0} \sin \left( \frac{\sin(x) - \frac{\pi}{2}}{\cos(x) - \tan(x)} \right)$$

(b) (10 points)

$$\lim_{t \rightarrow 3^+} \frac{t^2 - 5t + 6}{3 - t}$$

(c) (10 points)

$$\lim_{t \rightarrow 0} \cot(x) - \csc(x)$$

2. (a) (10 points) Let  $f(x) = \sin(x^3 + 3)$ . compute  $f'(x)$ .

(b) (10 points) Let  $g(x) = \frac{\sin(x) + \cos(x)}{x}$ . Compute  $g'(x)$ .

(c) (10 points) Let  $h(x) = \cos(3x)$ . Compute  $h^{(10)}(x)$ . [Note that you are asked to compute the tenth derivative of  $h$ ]

3. Consider the following equation:  $\sin(2y) + \cos(3x) = 0$ .

(a) (10 points) Show that  $(\frac{\pi}{4}, \frac{\pi}{8})$  lies on the graph of this equation.

(b) (20 points) Compute the tangent line to the graph at the point  $(\frac{\pi}{4}, \frac{\pi}{8})$ .

(c) (10 points) Explain why the graph of this equation cannot be the graph of a function  $y = f(x)$ .

4. Recall that  $\lim_{h \rightarrow 0} \frac{\sin(h)}{h} = 1$ .

(a) (20 points) Use the above limit to deduce that  $\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} = 0$ .

(b) (20 points) Use the formal definition of the derivative to show that if  $f(x) = \sin(2x)$  then  $f'(x) = 2 \cos(2x)$

5. Suppose that the temperature (in Fahrenheit) in Madison  $t$  hours after spring equinox is given by the function

$$T(t) = -10 \cos\left(\frac{\pi}{12}t\right) + 40 \sin\left(\frac{\pi}{12} \cdot \frac{t}{365}\right) + 40.$$

(a) (15 points) Is the function  $T$  periodic? If so, what is its period? If not, why not?

(b) (15 points) At what rate does the temperature change when  $t = 0$ ?

(c) (10 points) Is it morning, afternoon, evening or night when  $t = 0$ ? Briefly explain your answer.

6. (20 points) Let  $f(x) = \cos(2x)$ . Show that  $f(x) \cdot f'(x) = -\sin(4x)$ .

[You may use that  $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$  and  $\sin(u + v) = \cos(u)\sin(v) + \sin(u)\cos(v)$ .]