Math 171
Fall 2004

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# Exam 2

| Problem | Score |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Total | |

Name: _____

TA's Name _____

Section Time _____

No calculators, notes, or books are allowed. **You must show your work, and explain your reasoning to receive credit for your answers.**

Be sure to check your answers whenever possible.

On the last page of the exam, you will find lots of formulas and identities that may or may not be useful.

*Good luck!*

1. [5 points each] In each of the following, compute the limit or explain why it doesn't exist.

   (a) $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4}$

   (b) $\lim_{x \to \infty} \dfrac{x^3 + x - 1}{2x^3 + 3x}$

   (c) $\lim_{x \to 2^+} \dfrac{x - 4}{x - 2}$

2. [5 points] Let $f(x) = \begin{cases} 2x + a, & \text{if } x < 0 \\ -x^2 - 1, & \text{if } x \geq 0. \end{cases}$

   If $f$ is continuous, find $a$.

3. [5 points each] Differentiate each of the following functions.

   (a) $f(x) = 3x^7 - 5x^5 + 4x^{-1} - 1$

   (b) $g(x) = (x^2 - 1)^3(2 - 3x)^2$

   (c) $y = \dfrac{\sqrt{x^2 + 1}}{2x - 1}$

4. [10 points] Let $f(x) = x^2 + 2x$. Use the definition of the derivative to find $f'(-1)$.

5. [10 points] Find the equation of the tangent line to the curve $x^2 + xy + 2y^2 = 28$ at the point $(-2, -3)$.

6. Suppose $\tan\theta = -\frac{5}{12}$ and $\cos\theta < 0$. Find the following values. Be sure to justify your answers.

   (a) [7 points] $\sin\theta$

   (b) [3 points] $\tan(\theta + \pi)$

7. [5 points] Find the exact value of $\sin \frac{11\pi}{12}$.

8. [10 points] Find all solutions of $\sin 5x + \sin x = 0$.

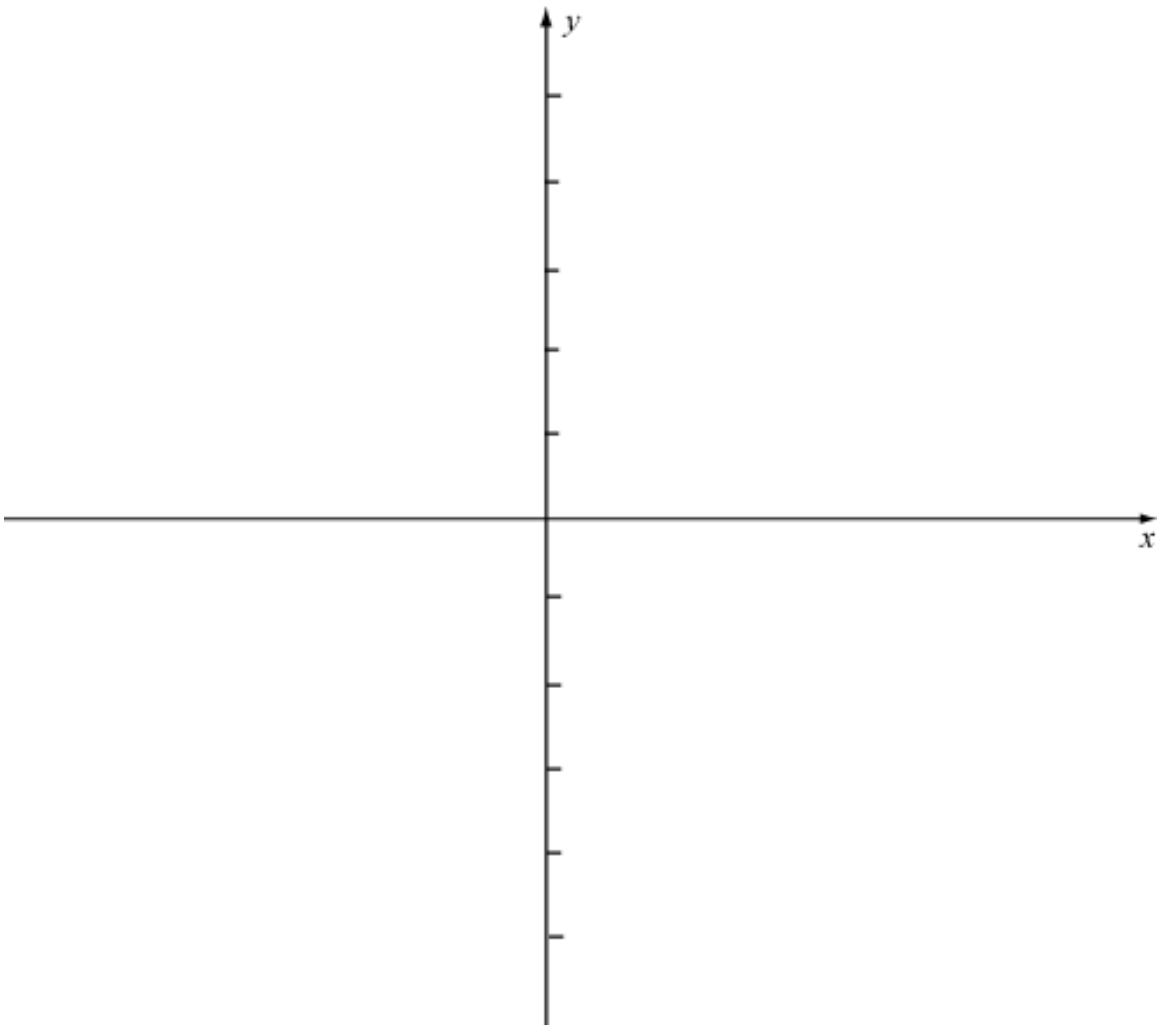
9. Let $f(x) = 2\cos\left(2x + \dfrac{\pi}{2}\right)$.

   (a) [3 points] Find the period, the amplitude, and the phase shift of $y = f(x)$.

   (b) [4 points] Find the $x$-intercepts of $y = f(x)$ in the interval $[-\pi, \pi]$.

9. (Continued)

   (c) [3 points] Sketch the graph of $y = 2\cos\left(2x + \frac{\pi}{2}\right)$. Label the $y$-axis, as well as the $x$-intercepts you found in part (b).



10. [10 points] You are watching a friend use a straw to drink juice out of a cone-shaped cup. You can see the height of the liquid drop and you measure the rate at which it drops to be a constant rate of $\frac{4}{3\pi}$ cm/sec. If the height of the cup is 10 cm and the diameter of the opening is 6 cm, what is the rate at which your friend is drinking his juice at the moment when the height of the liquid is 5 cm?

# Formulas You May or May Not Need

## Trig Formulas

*Co-Functions (Complements)*

$\cos(\frac{\pi}{2} - u) = \sin u$ $\quad$ $\tan(\frac{\pi}{2} - u) = \cot u$ $\quad$ $\sec(\frac{\pi}{2} - u) = \csc u$

$\sin(\frac{\pi}{2} - u) = \cos u$ $\quad$ $\cot(\frac{\pi}{2} - u) = \tan u$ $\quad$ $\csc(\frac{\pi}{2} - u) = \sec u$

*Addition & Subtraction*

$\cos(u + v) = \cos u \cos v - \sin u \sin v$ $\quad$ $\cos(u - v) = \cos u \cos v + \sin u \sin v$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$ $\quad$ $\sin(u - v) = \sin u \cos v - \cos u \sin v$

$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ $\quad$ $\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

*Double-Angle & Half-Angle*

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$

$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$

$\sin^2 \frac{v}{2} = \frac{1 - \cos v}{2}$

$\cos^2 \frac{v}{2} = \frac{1 + \cos v}{2}$

$\tan^2 \frac{v}{2} = \frac{1 - \cos v}{1 + \cos v}$

$$\tan \frac{v}{2} = \pm\sqrt{\frac{1 - \cos v}{1 + \cos v}} = \frac{1 - \cos v}{\sin v} = \frac{\sin v}{1 + \cos v}$$

*Sums & Products*

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$ $\quad$ $\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$ $\quad$ $\sin a - \sin b = 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$ $\quad$ $\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$ $\quad$ $\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$

## Geometric Formulas

*volume of a cone:* $V = \frac{1}{3}\pi r^2 h$ $\quad$ *volume of a sphere:* $V = \frac{4}{3}\pi r^3$

*surface area of a cone:* $S = \pi r \sqrt{r^2 + h^2}$ $\quad$ *surface area of a sphere:* $S = 4\pi r^2$