Lec. 2
Fall 2004

C. Gómez
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Math 171
Final Examination

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Name: _____

TA's Name _____

Section Time _____

No calculators, notes, or books are allowed. **You must show your work, and explain your reasoning to receive credit for your answers.** Be sure to check your answers whenever possible!

On the last page of the exam, you will find lots of formulas and identities that may or may not be useful.

Good luck!

1. [5 points each] Solve each of the following equations.

   (a) $x + \sqrt{5x + 19} = -1$

   (b) $\cos 2x - \tan x = 1$

2. [5 points each] Compute the limit, or explain why it doesn't exist. In particular, if it is unbounded, determine whether it is negative or positive infinity.

   (a) $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

   (b) $\lim_{x \to 0} \dfrac{3x \tan x}{\sin x}$

   (c) $\lim_{t \to -\infty} \dfrac{-3t^3}{2t^3 + 5}$

3. [5 points each] Differentiate each of the following functions. Don't simplify your answers.

   (a) $f(x) = \sin x \tan x$

   (b) $g(x) = (\sqrt{1+x^2}+1)^2$

   (c) $f(x) = \left(\dfrac{\sin x}{x+1}\right)^3$

4. [5 points] Give the precise (using ϵ and δ) definition of

$$\lim_{x \to -\frac{1}{2}} (2x - 5) = -6$$

You don't have to prove the statement!

5. (a) [3 points]Find the amplitude, the period, and the phase shift of $y = -2\cos(\frac{\pi}{4}x - \frac{\pi}{2})$.

   amplitude:

   period:

   phase shift:

   (b) [7 points] Sketch the graph of $y = -2\cos(\frac{\pi}{4}x - \frac{\pi}{2})$, labeling the x-intercepts in one phase of the function.

6. [10 points] Next year I need to put in a rectangular garden that is 200 square feet in area. Last summer the rabbits ate everything, so I'm going to put a fence around my new garden. Find the dimensions of my garden that will require the least amount of fencing if one side of the garden will already be protected by the garage.

7. [10 points each] Carefully sketch each of the following graphs. Label any and all intercepts, asymptotes, extrema, and points of inflection.

   (a) $f(x) = x^4 + 4x^3$

7. *(continued)*

   (b) $f(x) = \dfrac{x}{(x+2)^2}$

8. [5 points] The position of an object moving along a horizontal axis is given by $s(t) = 5t^4 - t^5$, where $t > 0$. When is the object speeding up?

9. [5 points] Suppose a spherical snow ball is melting and its radius is decreasing at a constant rate, changing from 12 inches to 8 inches in 30 minutes. How fast is the volume changing when the radius is 10 inches?

10. [5 points] Does the function $f(x) = x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$ satisfy the conditions of the Mean Value Theorem on the interval $[0, 8]$?

    If not, explain why not.

    If it does, then state those conditions and find all numbers c in the interval $[0, 8]$ for which $f'(c) = \dfrac{f(8) - f(0)}{8 - 0}$.

Trig Formulas

Co-Functions (Complements)

$\cos(\frac{\pi}{2} - u) = \sin u$ $\quad$ $\tan(\frac{\pi}{2} - u) = \cot u$ $\quad$ $\sec(\frac{\pi}{2} - u) = \csc u$

$\sin(\frac{\pi}{2} - u) = \cos u$ $\quad$ $\cot(\frac{\pi}{2} - u) = \tan u$ $\quad$ $\csc(\frac{\pi}{2} - u) = \sec u$

Addition & Subtraction

$\cos(u + v) = \cos u \cos v - \sin u \sin v$ $\quad$ $\cos(u - v) = \cos u \cos v + \sin u \sin v$

$\sin(u + v) = \sin u \cos v + \cos u \sin v$ $\quad$ $\sin(u - v) = \sin u \cos v - \cos u \sin v$

$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ $\quad$ $\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

Double-Angle & Half-Angle

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 1 - 2\sin^2 u$
$= 2\cos^2 u - 1$

$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$

$\sin^2 \frac{v}{2} = \frac{1 - \cos v}{2}$

$\cos^2 \frac{v}{2} = \frac{1 + \cos v}{2}$

$\tan^2 \frac{v}{2} = \frac{1 - \cos v}{1 + \cos v}$

$$\tan \frac{v}{2} = \pm\sqrt{\frac{1 - \cos v}{1 + \cos v}} = \frac{1 - \cos v}{\sin v} = \frac{\sin v}{1 + \cos v}$$

Sums & Products

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$ $\quad$ $\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$ $\quad$ $\sin a - \sin b = 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$ $\quad$ $\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$

$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$ $\quad$ $\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$

Geometric Formulas

area of a circular sector: $A = \frac{1}{2}r^2\theta$
length of a circular arc: $s = r\theta$

volume of a sphere: $V = \frac{4}{3}\pi r^3$
surface area of a sphere: $S = 4\pi r^2$

volume of a cone: $V = \frac{1}{3}\pi r^2 h$
surface area of a cone: $S = \pi r\sqrt{r^2 + h^2}$

volume of a circular cylinder: $V = \pi r^2 h$
surface area of a circular cylinder: $S = 2\pi rh$