Math 171 | C. Gómez
Fall 2005 | December 20, 2005

Final Exam

Name: ________________________________

Problem	Score
1	
2	
3	
4	
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6	
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8	
9	
10	
Total	

Please circle your TA's name:

Sharon Garthwaite

Evan (Alec) Johnson

Paul Johnson

Patrick Rault

This exam contains 13 pages, and 10 problems. Before you begin, please make sure all the pages are here.

The last page of the exam contains some formulas that may or may not come in handy. If you decide to tear off that page, please do so carefully.

No calculators, notes, or books are allowed.

You must show all your work, and explain your reasoning to receive credit for your answers.

Be sure to check your answers whenever possible.

Good luck!

1. (a) [6 points] Find an equation for the line perpendicular to $2x + 3y + 4 = 0$ and containing the point $(-1, 2)$.

   (b) [6 points] Find an equation for the parabola with vertex $(3, -1)$, and containing the point $(1, -5)$.

2. Evaluate the limit.

   (a) [4 points] $\displaystyle\lim_{x\to -2^-} \frac{x+1}{x^2-4}$

   (b) [4 points] $\displaystyle\lim_{x\to 0} \frac{\tan x}{x}$

   (c) [4 points] $\displaystyle\lim_{\theta\to \frac{\pi}{4}} \frac{1-\tan\theta}{\sin\theta - \cos\theta}$

3. [12 points] Find an equation for the tangent line to the curve

$$y + \cos(xy^2) + 3x^2 = 4$$

at the point $(1, 0)$.

4. [12 points] A farmer in Elmwood (the official "UFO Capital of Wisconsin") observes something flying at a constant speed, at an altitude of 1 mile, headed straight for a point directly over his head. He notes that when the angle of elevation of his line of sight (from the horizontal) is $\frac{\pi}{6}$ radians, it is increasing at a rate of 2 radians per minute. What is the speed of the UFO?

5. (a) [4 points] Let $f(x) = \frac{x-3}{x^2-1}$. Find the intervals on which $f(x)$ is positive and the intervals on which it is negative.

(b) [4 points] Find the horizontal and vertical asymptotes (if any) of $f(x) = \frac{x-3}{x^2-1}$.

5. (continued)

   (c) [4 points] Sketch the graph of $f(x) = \frac{x-3}{x^2-1}$, labeling all asymptotes and intercepts.

6. (a) [3 points] Find the amplitude, the period, and the phase shift of $y = -\sqrt{2}\sin(\frac{\pi}{2}x - \frac{\pi}{4})$.

   amplitude:

   period:

   phase shift:

   (b) [9 points] Sketch the graph of $y = -\sqrt{2}\sin(\frac{\pi}{2}x - \frac{\pi}{4})$, labeling the x-intercepts in one period, and the maximum and minimum values of the function on the y-axis.

7. Differentiate each of the following functions. Don't simplify your answer.

   (a) [4 points] $f(x) = \dfrac{x^2 + x}{(2x-1)^2}$

   (b) [4 points] $y = \sin(\cos 3x)$

   (c) [4 points] $g(t) = \tan^2(2t^2 + t - 1)$

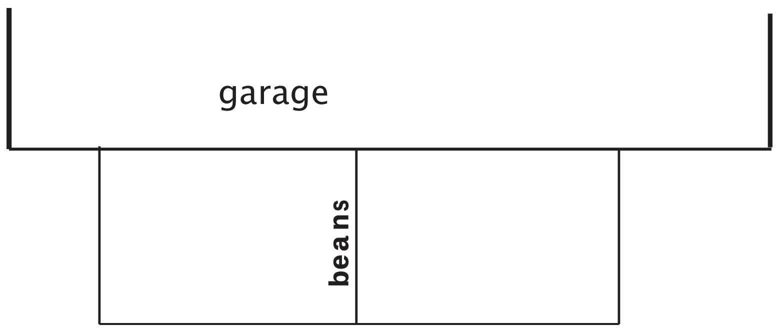
8. [12 points] Find the center and radius of the circle with equation

$$4x^2 + 4y^2 + 24x - 16y + 3 = 0$$

center: ____________ **radius:** ____________

9. [12 points] Find the exact value of $\sin 2\theta$ if $\pi < \theta < \frac{3\pi}{2}$ and $\sin\theta = -\frac{4}{5}$.

10. [12 points] Next year I need to put a fence around my rectangular vegetable garden to keep the rabbits out. I also need to put a fence in the middle of the garden parallel to one of the sides, so that the beans will have some support. Since I only have 60 feet of fencing, I'm going to use the garage as one side of the garden. Find the dimensions of the garden with maximum possible area.



Trig Formulas

Co-Functions (Complements)

$\cos(\frac{\pi}{2} - u) = \sin u$ $\quad$ $\tan(\frac{\pi}{2} - u) = \cot u$ $\quad$ $\sec(\frac{\pi}{2} - u) = \csc u$

$\sin(\frac{\pi}{2} - u) = \cos u$ $\quad$ $\cot(\frac{\pi}{2} - u) = \tan u$ $\quad$ $\csc(\frac{\pi}{2} - u) = \sec u$

Addition & Subtraction

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

Double-Angle & Half-Angle

$\sin 2u = 2 \sin u \cos u$

$$\cos 2u = \cos^2 u - \sin^2 u = 1 - 2\sin^2 u = 2\cos^2 u - 1$$

$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$

$\sin^2 \frac{v}{2} = \frac{1 - \cos v}{2}$

$\cos^2 \frac{v}{2} = \frac{1 + \cos v}{2}$

$\tan^2 \frac{v}{2} = \frac{1 - \cos v}{1 + \cos v}$

$$\tan \frac{v}{2} = \pm\sqrt{\frac{1 - \cos v}{1 + \cos v}} = \frac{1 - \cos v}{\sin v} = \frac{\sin v}{1 + \cos v}$$

Sums & Products

$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$

$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$

$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$

$\sin a - \sin b = 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$

$\cos a + \cos b = 2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$

$\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$

Geometric Formulas

length of a circular arc: $s = r\theta$
area of a circular sector: $A = \frac{1}{2}r^2\theta$

volume of a cone: $V = \frac{1}{3}\pi r^2 h$
surface area of a cone: $S = \pi r\sqrt{r^2 + h^2}$

volume of a sphere: $V = \frac{4}{3}\pi r^3$
surface area of a sphere: $S = 4\pi r^2$

volume of a right circular cylinder:
$V = \pi r^2 h$
surface area of a right circular cylinder:
$S = 2\pi rh$