

Exam 3

Name: _____

TA's Name _____

Section Time _____

Problem	Score
1	
2	
3	
4	
5	
Total	

No calculators, notes, or books are allowed. You must show all your work, and explain your reasoning to receive credit for your answers.

Work the problems in an order that will maximize your score, and check your answers whenever possible.

Good luck!

1. [15 points] Find the equation of the tangent line to the curve

$$\frac{y}{y^2 + 1} + x = 1$$

at the point $(1, 0)$.

2. [20 points] Suppose that the air is being let out of a spherical balloon, so that its radius is decreasing at 10 inches per minute. How fast is the surface area of the balloon changing when the radius is 2? (Note: the surface area of a sphere of radius r is $4\pi r^2$).

3. (a) [15 points] Find the amplitude, period, and phase shift and sketch the graph of the function

$$f(x) = -2 \cos \left(\frac{1}{2}x + \frac{\pi}{4} \right).$$

Be sure to include at least two full cycles of the function.

(b) [10 points] Find all solutions to the equation

$$-2 \cos\left(\frac{1}{2}x + \frac{\pi}{4}\right) = 2$$

(Note: you must solve the equation rather than read the answers off the graph from part (a), although you can use the graph to check your answer.)

4. Verify the following identities:

(a) [10 points] $\csc^2 x - \cot^2 x = 1$.

(b) [10 points] $\cos^4 x + \sin^2 x = \cos^2 x + \sin^4 x$.

5. [20 points] Find all solutions of the following equation:

$$\cos 4t \cos 2t = \frac{\sqrt{3}}{2} - \sin 4t \cos 2t.$$

(Hints: Use the subtraction law for cosine. Cosines of some common reference angles are $\cos \frac{\pi}{3} = \frac{1}{2}$, $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, and $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$.)