

Midterm, Mathematics 171, Fall 2001, Ernesto Lupercio
No books are allowed.

Name :

1 2 3 4 5 6 Total /100.

1. Find $\frac{dy}{dx}$ for the following expressions:

(5 points) a) $y = \cos(x^2)$.

(10 points) b) $y = \cos(\sin(x \sin(x)))$.

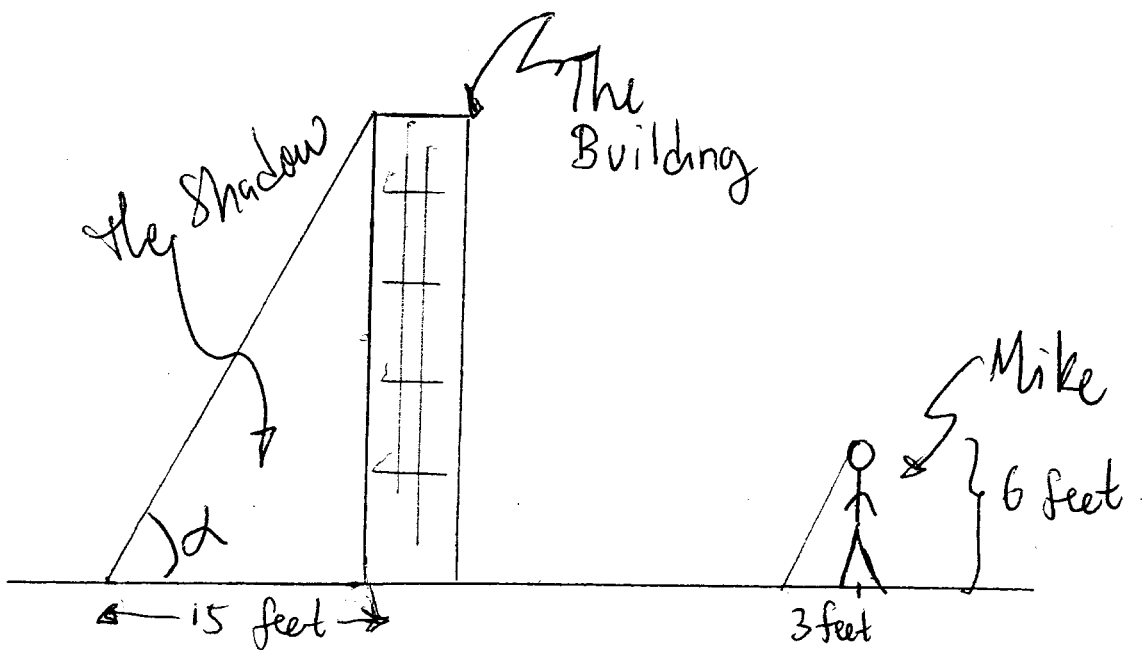
(10 points) c) $x = 5 \cos(2\pi t) \quad y = 5 \sin(2\pi t)$.

(15 points) d) $y \cos(x) - x \sin(2y) = 7$.

2. Suppose that at 3:00 PM last Saturday you measured Mike's shadow and it was 3 feet long. Suppose that Mike are 6 feet tall.

(15 points) a) Find the angle α that the shadow of the building depicted below makes with the ground.

(10 points) b) Find the height of the building if you know that its shadow is 15 feet long.



3. For the following functions find the points of discontinuity and decide if they are removable or non-removable singularities.

(10 points) a) $f(x) = \frac{x+1}{x^2-x-2}$.

(10 points) b) $f(x) = \frac{\sin(5x-15)}{x-3}$.

(10 points) c) $f(x) = \frac{\tan(x)}{x}$.

4. For a trigonometric function of the form

$$y = a \sin(k(x - b))$$

we say that a is the amplitude, b is the phase shift and $2\pi/k$ is the period.

Consider the function

$$f(x) = \frac{\sqrt{3}}{2} \sin\left(2x - \frac{\pi}{3}\right) - \frac{1}{2} \cos\left(2x - \frac{\pi}{3}\right)$$

(5 points) a) Find $f'(x)$

(10 points) b) Find all the critical points.

(5 points) c) Find all the maxima of $f(x)$.

(10 points) d) Find the amplitude and the period of $f(x)$.

(5 points) e) Find the phase shift.

(10 points) f) Sketch the graph of $f(x)$ indicating clearly the scales on the axis, the maxima, minima, period, phase shift and the amplitude.

(20 points) 5. Bart Simpson is blowing a balloon. If in the last 2 seconds the volume of the balloon grew 1 percent, approximately how much did the radius grow (in percentage)? You must use the differential, or the linear approximation formula to solve this problem.

6. Consider the cubic polynomial function $f(x) = x^3 - x^2 - 4x + 4$.
- a) (10 points) Compute the slope $m(x_0)$ at the point $(x_0, f(x_0))$.
 - b) (10 points) Sketch the graph of $y = f(x)$ using the method described in class. Label carefully the scales on the axis, and the relevant points on the graph.
 - c) (5 points) Where are the local maximum and the local minimum for this function?