

FINAL EXAM, Mathematics 171, Fall 2001, Ernesto Lupercio  
No books are allowed.

Name :

Instructor:

Phone number:

e-mail :

1	2	3	4	5	6	7	8	9	Total	/370.
---	---	---	---	---	---	---	---	---	-------	-------

***IMPORTANT:***

1) You **must** show all your work. Just writing an answer in a problem will not be considered enough for partial credit.

2) The use of calculators is allowed, except for the use of graphic capabilities. The use of graphic functions or features in your calculator **not** allowed.

3) You may use both sides of the paper if you do not have enough space to finish your work on one side.

4) Make sure that you have a total of **10** pages.

1. A window has the shape of a rectangle of base  $x$  and side  $s$  topped by a semicircle. The total perimeter of the window is  $90ft$ .
- a) (10 points) Find a formula for  $s$  in terms of  $x$ .
  - b) (10 points) Find a formula for the area of the window in terms of  $x$ .
  - c) (20 points) Find the maximum possible area of the window. For what values of  $x$  and  $s$  is this maximum attained?

2.(15 points) a) Using the linear approximation formula approximate

$$8.2^{\frac{1}{3}}$$

(15 points) b) Consider a square of area  $A = 4 \text{ ft}^2$  and side  $l = 3 \text{ ft}$ . If the area of the square changed to 4.1 approximate the change  $\Delta l$  of the side using the linear approximation formula.

3. a) (5 points) What is the slope of the line  $L: 2x + y = 5$  ?
- b) (10 points) Let  $L'$  be a line passing through the origin and perpendicular to  $L$ . Find its equation.
- c) (10 points) Find the intersection point  $P$  of  $L$  and  $L'$ .
- d) (5 points) Find the distance from  $(0,0)$  to  $P$ . (We call this distance the distance from  $(0,0)$  to the line  $L$ .)

4. For a trigonometric function of the form

$$y = a \sin(k(x - b))$$

we say that  $a$  is the amplitude,  $b$  is the phase shift and  $2\pi/k$  is the period.

Consider the function

$$f(x) = \frac{\sqrt{3}}{2} \sin\left(2x - \frac{\pi}{3}\right) - \frac{1}{2} \cos\left(2x - \frac{\pi}{3}\right)$$

(10 points) a) Find  $f'(x)$

(10 points) b) Find all the critical points.

(5 points) c) Find all the maxima of  $f(x)$ .

(10 points) d) Find the amplitude and the period of  $f(x)$ .

(5 points) e) Find the phase shift.

(10 points) f) Sketch the graph of  $f(x)$  indicating clearly the scales on the axis, the maxima, minima, period, phase shift and the amplitude.

5. Find  $\frac{dy}{dx}$  for the following expressions:

(15 points) a)  $y = \cos(x^3)$ .

(15 points) b)  $y = \sin(x \sin(x))$ .

(15 points) c)  $x = 4 \cos(\pi t)$      $y = 3 \sin(\pi t)$ .

(15 points) d)  $y \cos(x) - x \sin(3y) = 9$ .

6. Consider the cubic polynomial function  $f(x) = -6 + 11x - 6x^2 + x^3$ .
- a) (10 points) Compute the slope  $m(x_0)$  at the point  $(x_0, f(x_0))$ .
  - b) (10 points) Sketch the graph of  $y = f(x)$  using the method described in class. Label carefully the scales on the axis, and the relevant points on the graph.
  - c) (10 points) Where are the local maximum and the local minimum for this function?

7. For the following functions find the points of discontinuity and decide if they are removable or non-removable singularities.

(10 points) a)  $f(x) = \frac{x+1}{2-x-2x^2+x^3}$ .

(10 points) b)  $f(x) = \frac{\sin(7x-21)}{x-3}$ .

(10 points) c)  $f(x) = \frac{\tan(x)}{x}$ .

(50 points) 8. How many solutions has the equation

$$2 - 9x + 7x^2 + 6x^3 = 0$$

in the interval  $0 \leq x \leq 1$  ?

(50 points) 9. Ellen want to build a rectangular fence with 3 sides; the place of the fourth side will be taken by the Cam river. With the materials she has the total length of the fence will be 100 ft. What is the maximum possible area that the fence can bound?