

Math 171 – Lecture 1, 2nd Midterm

Nov. 7, 2002

Instructor: Manuel Portilheiro

Your Name: _____

Your section:(or TA) _____

Question 1: _____

Question 2: _____

Question 3: _____

Question 4: _____

Question 5: _____

Question 6: _____

Total: _____

Instructions

1. No books or notes of any kind are allowed.
2. There are six questions on this exam. Answer all questions.
3. Show all your work, and justify your answers.
4. This exam is worth 25% of your grade.

Question 1: Consider the function

$$g(x) = \frac{x}{x^2 - 9}.$$

- (a) What is the domain of g ?
- (b) Compute the limits of g at infinity and the one-sided limits at the points not in its domain.
- (c) What are the vertical and horizontal asymptotes of g ?
- (d) Based on (c) make a sketch of the graph of g .
- (e) Is g one-to-one? Why?

Question 2: Find the equation of the straight line tangent to the graph of the function h at the point $(x, y) = (1, 8)$, with

$$h(x) = \frac{(x^2 + 1)^3}{\sqrt{x}}.$$

Question 3: State the Chain Rule for differentiating the composition of two functions. Use it to differentiate $f \circ g$ with

$$f(x) = x^5 + x^6 \quad \text{and} \quad g(x) = 3x^2 + 1.$$

Question 4: Verify the following identities:

$$(a) \sec \theta (\cos \theta - \sin \theta) = 1 - \tan \theta.$$

$$(b) \sec \alpha + \tan \alpha = \frac{1}{\sec \alpha - \tan \alpha}.$$

Question 5: Sketch the graph of the function

$$f(x) = 2 \cos(2x)$$

on the interval $[-\pi, \pi]$. What is the **period** of f ? What is the **amplitude** of f ?

Question 6: Consider the function $r(x)$ defined by

$$r(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ x & \text{if } 0 < x < \pi, \\ \pi \cos x & \text{if } x \geq \pi. \end{cases}$$

- (a) Sketch the graph of r on the interval $[-\pi, \pi]$.
(b) Is r continuous at $x = 0$? At $x = \pi$?