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Math 171 – Lecture 1, **Final Exam**
Instructor: Manuel Portilheiro Dec. 20th, 2002

Your Name: _____
Your section:(or TA) _____

Question 1: _____

Question 2: _____

Question 3: _____

Question 4: _____

Question 5: _____

Question 6: _____

Question 7: _____

Question 8: _____

Total: _____

Instructions

1. No books or notes of any kind are allowed.
2. There are eight questions on this exam. Answer all questions.
3. Show all your work, and justify your answers.
4. This exam is worth 50% of your grade.

Question 1: Consider the function

$$g(x) = \begin{cases} 4x - x^2 - 2, & \text{if } 1 \leq x < 3; \\ \sec(x-3), & \text{if } 3 \leq x \leq 3 + \frac{\pi}{4}. \end{cases}$$

(a) Find all critical values of g.

(b) Determine the local and global maxima and minima of g.

Question 2: Let

$$f(x) = \sqrt{1 + \tan^2(x)} \quad \text{and} \quad g(x) = \sin^2\left(\frac{\pi}{4}x + \frac{\pi}{4}\right).$$

(a) Compute $f'(x)$ and $g'(x)$.
(b) Solve the equations

$$f(x) = 2; \qquad g(x) = \frac{1}{2}.$$

Question 3: Simplify the expressions

(a) $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$,

(b) $\arcsin\left(\cos\left(\frac{2\pi}{3}\right)\right)$,

(c) $\cos\left(\arctan\left(x\right)\right)$,

(d) $\cos\left(2\arcsin\left(x\right)\right)$.

Question 4: Sketch the graph of a continuous function f defined on $[-5, 0)$ such that:
- $f(-5) = 1$, $f(-4) = 0$, $f(-3) = -1$, $f(-2) = 2$, $f(-1) = 3$ and $\lim_{x \to 0^-} f(x) = -\infty$;
- $f'(x) < 0$ on $(-5, -3)$ and $(-1, 0)$; $f'(x) > 0$ on $(-3, -1)$;
- $f''(x) < 0$ on $(-5, -4)$ and $(-2, 0)$; $f''(x) > 0$ on $(-4, -2)$.

Question 5: Consider the function

$$h(x) = x\sqrt{x^2 - 4}.$$

(a) Determine the domain of h.
(b) Compute the limits of $h(x)$ as $x \to +\infty$, $x \to -\infty$, $x \to -2^-$ and $x \to 2^+$.

Question 6: Graph the function

$$f(x) = \frac{1}{1+x^2}.$$

Remeber to pay attention to the domain, parity, intervals of monotonicity, concavity, local and global extrema, inflexion points and asymptotes.

Question 7: Verify the following identities:

$$\text{(a)}\ \frac{\cot\theta - \tan\theta}{\sin\theta + \cos\theta} = \csc\theta - \sec\theta;$$

$$\text{(b)}\ \frac{1}{\tan\alpha + \tan\beta} = \frac{\cos\alpha\cos\beta}{\sin(\alpha+\beta)}.$$

Question 8: Let $f(x) = \tan(x)$.

(a) Show that

$$\frac{f(x+h) - f(x)}{h} = (1 + \tan^2(x))\frac{\tan h}{h}.$$

(b) Compute $f'(x)$ using the definition of derivative.
(Hint: You can use the fact that

$$\lim_{x \to 0} \frac{\sin(h)}{h} = 1$$

without proving it.)