

MATH 210  
Test 3

(P. Ney)  
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Name: \_\_\_\_\_

ID#: \_\_\_\_\_

Discussion Section: (day, time): \_\_\_\_\_

PROBLEM	VALUE	SCORE
I	20	
II	24	
III	16	
IV	20	
V	20	
TOTAL	100	

I. (20 pts) A mouse moves in an experimental maze of three rooms, labelled 1, 2, 3. At the end of each minute a bell rings and he makes a move.

If he is in room 1 when the bell rings, he moves to 2.

If he is in 2, he moves to 3.

If he is in room 3 when the bell rings, he is sent to room 1 with probability  $\frac{1}{2}$ , and he stays in room 3 for another minute with probability  $\frac{1}{2}$ .

This procedure is repeated. Let  $X_n$  denote his location at time  $n$ . Then  $X_0, X_1, \dots$  is a Markov chain.

(a) Draw the transition diagram and identify the transition matrix for this Markov chain.

(b) If the mouse starts at time 0 in room 2, what is the probability that it is in room 3 after 4 minutes.

(c) What is

$$P\{X_2 = 3, X_3 = 3, X_4 = 3 \mid X_0 = 2\}?$$

II. (24 pts) Let  $X_0, X_1, X_2, \dots$  be a Markov chain with state space consisting of the two states 0 and 1, and with transition matrix

$$\begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix} \end{array}$$

Assume that  $X_0 = 0$ . Find

(a)  $P(X_4 = 0)$ .

(b)  $P(X_1 = 0 \text{ and } X_3 = 0)$ .

(c)  $EX_2$ .

(d)  $P(X_1 = X_3)$ .

(16 pts) Let  $X_0, X_1, X_2, \dots$  be a Markov chain with the same state space and transition matrix as in problem II, namely

$$P = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix} \end{matrix}.$$

However now instead of taking  $X_0 = 0$ , start with an initial state vector  $V^{(0)} = \left[\frac{1}{4}, \frac{3}{4}\right]$ .

(a) Find  $P(X_4 = 0)$ .

(b) Find variance  $(X_2)$ .

(20 pts) Let  $X_0, X_1, \dots$  be a Markov chain with state  $\{0, 1, 2, \}$  and transition matrix

$$\begin{array}{c} 0 \quad 1 \quad 2 \\ \begin{array}{l} 0 \\ 1 \\ 2 \end{array} \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \end{array}$$

Find (approximately)

(a)  $P(X_{25} = 1 \mid X_0 = 1)$ .

(b)  $P(X_{25} = X_{27} = 1 \mid X_0 = 1)$ .

V. (20 pts)

- (a) You put \$1000 in a savings account that brings interest of 8% annually, compounded quarterly. After forgetting about the account you return to it after 10 years. What is it worth at that time?
- (b) John is borrowing an amount of  $P$  dollars from his uncle to help pay for tuition. He plans to repay \$1000 at the end of one year, and \$1000 at the end of two years. If the loan is calculated at an annual interest rate of 8%, compounded quarterly, what should  $P$  be?