

Math 210 (J. Pakianathan)

Exam # 3, April 10, 2000
100 points

BE SURE TO SHOW YOUR WORK FOR FULL CREDIT!

Calculators and a 3"x5" notecard are allowed in this exam.

NAME:

DISCUSSION SECTION:

Scores: (for grader's use only).

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. (20 points) Solve the following system of linear equations for x, y, z by using the Gauss-Jordan "row reduction" method. You must show your work for full credit.

$$x + 3z = 2$$

$$2x + 4y = 4$$

$$x + 2y + 2z = 5$$

$$4x + 6y + 5z = 11$$

y
k

2. (24 points)

Find all solutions to the following systems of linear equations using the Gauss-Jordan method. Show your work - an answer with no correct supporting argument or work will not get full credit.

[12 pts] (a)

$$2w + 2x + 8y + 4z = 1$$

$$2y + 4z = 4$$

[12 pts] (b)

$$4x + 2y = 4$$

$$3x + 3y = 5$$

$$3x + y = 6$$

3. (26 points)

[13 pts] (a) Given

$$\mathbf{A} = \begin{pmatrix} 1 & a & b^2 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

where a and b are unknown constants. Find \mathbf{A}^{-1} .

[13 pts] (b) Now let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}$. Find

$$3\mathbf{A}^{-1} - \mathbf{B}^2.$$

(Recall $\mathbf{B}^2 = \mathbf{B}\mathbf{B}$.)

4. (30 points) You are given the following linear programming problem: The "Nut Factory" produces a mixture of peanuts and cashews. The company guarantees that at least 40% of the total weight of the mixture is cashews. Due to a contract, it has to produce at least 1000 pounds or more of the mixture. The peanuts cost \$0.80 per pound and the cashews cost \$1.50 per pound. Find the amount of each kind of nut the company should use to fulfill its contract with minimal cost if it has only 720 pounds of peanuts available and 1000 pounds of cashews available.

We will solve this problem in steps. Let x be the number of pounds of peanuts to use in the mixture and y be the number of pounds of cashews. Then the constraints of the problem above translate as:

$$\begin{aligned}x + y &\geq 1000 \\y &\geq .4(x + y) \\y &\leq 1000 \\x &\leq 720 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

[3 pts] (a) Near each constraint above, indicate whether the constraint comes from the limited peanuts available, the limited cashews available, the contract requirement, the company guarantee on cashew content in the mixture or other constraint. If you answer other, you will need to explain in part (b).

[2pts] (b) Explain (a sentence explanation is fine) where the 'other' constraints in (a) come from.

[5pts] (c) Write down a function for the cost C in terms of x and y .

[15 pts] (d) Sketch the feasible set for the constraints above, labeling all lines and corner points. (You might find it convenient to rewrite some of the inequalities first by bringing all the variables to the left hand side.)

[5 pts] (e) Solve the initial problem, i.e., for what values of x and y is C minimized subject to the constraints?