Math 211 Spring 2007

Final Exam S. Bolotin

Your Name: ______________________________

Your TA: ______________________________

PROBLEM	POINTS	SCORE
I	20	
II	20	
III	20	
IV	20	
V	20	
VI	20	
VII	20	
VIII	20	
IX	20	
X	20	
TOTAL	200	

Show all your work: no work - no credit. Leave your answers in exact form (using e, $\ln 2$, $\sqrt{2}$ and similar numbers). Circle your answer. Hand in your exam, together with the formula sheet, to your TA.

I. Find the derivatives of the following functions.

(a) $f(x) = \cos(\ln x)$

(b) $f(x) = \ln(x + \sqrt{x^2 - 1})$

(c) $f(x) = \dfrac{\sin^2 x}{\cos x}$

II. Find the integrals

(a) $\displaystyle\int \frac{x+1}{x^2+2x}\,dx$

(b) $\displaystyle\int xe^{-x}\,dx$

(c) $\displaystyle\int_0^{\pi/2} \frac{dx}{\cos^2(x/2)}$

III. For the function $f(x) = 2x^3 - 9x^2 - 60x + 10$ find

(a) intervals on which $f(x)$ is increasing (decreasing).

(b) Intervals on which $f(x)$ is concave up (down).

(c) Find global maximum and minimum values of $f(x)$ on the interval $[0, 4]$.

IV. Find the solution of the differential equation $\frac{dy}{dx} = \frac{1+x}{xy}$, $x > 0$, with the initial condition $y(1) = -4$.

V. Let $f(x, y) = e^x \tan(x - y)$.

(a) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

(b) Find the tangent plane to the graph $z = e^x \tan(x - y)$ at the point $(1, 1, 0)$.

VI. Find the critical points of the function $f(x, y) = y\sqrt{x} - y^2 - x + 6y$ and identify them as local minimum, local maximum, or saddle points.

VII. Equation $\cos(x - y) = y \sin x$ defines $y = y(x)$ as a function of x. Find $\dfrac{dy}{dx}$ at $x = \pi/2$, $y = 0$.

VIII. Find the area between the curves $y = \sqrt{x-1}$ and $y = (x+1)/3$ for $1 \leq x \leq 3$.

IX. A company manufactures two products. The price p and demand x for the first product are related as $p = 16 - x$, and price q and demand y for the second product are related as $q = 19 - y/2$. If the cost function is $C(x, y) = 10x + 12y - xy + 6$, find the prices p and q that maximize profit.

X. Use the Lagrange multipliers method to find maximal and minimal values of the function $f(x, y) = xy$ subject to the constraint $9x^2 + y^2 = 4$.