Math 211 – FINAL EXAM – Fall 2007

YOUR NAME: ______________________________

Circle your TA's name:

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Show all your work. Answers without showing all work receive zero points.

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1. *(10 points)* Compute the following:

(a) Consider $f(x) = \frac{1}{2-x}$ and compute $f'(x)$ *using the definition of the derivative.*

(b) $\displaystyle\lim_{x\to-1} \frac{x+1}{x^3+1}$

(c) $\displaystyle\lim_{n\to\infty} 40\left(1+\frac{3}{n}\right)^{5n}$

(d) $\displaystyle\lim_{t\to-\infty} \frac{t^3+2t}{t^2-t+1}$

2. *(10 points)* Is it possible to extend the definition of $h(t) = \frac{2t}{(t-1)^2-1}$ to make it continuous at $t=0$? Explain your answer.

3. *(10 points)* The mass of a piece of radioactive substance decays exponentially. Initially it was 600 mg, and after 2 hours it is 400 mg. How long will it take until only 200 mg are left?

4. *(10 points)*

(a) Express $\log_3 \sqrt{27}$ and $\log_4 8$ as an integer or fraction without using a calculator. What general formula did you use?

(b) Solve the equation $\ln(2x + 1) = 4$.

5. *(10 points)* Find the equation of the tangent line to the graph of $y = x^4$ at the point where the slope equals 4.

6. *(10 points)* A particle is moving along the y-axis with $y(t)$ being its coordinate at time t. Represent each of the following statements by a pair of inequalities involving $\frac{dy}{dt}$ and $\frac{d^2y}{d^2t}$:

(a) The particle is moving up with increasing speed.

(b) The particle is moving up with decreasing speed.

(c) The particle is moving down with increasing speed.

7. *(10 points)*
(a) Compute the derivative of $f(x) = x^3 \ln \frac{1}{2x}$.

(b) Compute the derivative of $g(t) = \frac{e^{\ln t} + e^t - \ln(e^t)}{t}$.

(c) Compute the derivative of $h(x) = \ln(e^{\frac{1}{x}} + 1)$.

8. *(15 points)* Cars A and B start from the same intersection, but not at the same time. Car A starts first and heads north at constant speed of 50 mi/h. One hour later car B starts and heads east at 60 mi/h. Write down a formula for the rate of change of the distance between the two cars (after car B left the intersection).

9. *(10 points)* Consider the function $f(x) = |x - 1|$. Find its critical points and its derivative; use its derivative to determine where it is increasing, where it is decreasing, and where it has a local maximum and minimum, if any. Then, plot the graph of $f(x)$.

14. *(10 points)* Compute the area of the region enclosed by the curves $y = 2x^2+2x+4$ and $y = -x^2+5x+10$ between their intersection points.

15. *(10 points)* If the demand curve is $D(q) = -0.4q + 21$ and the supply curve is $S(q) = 0.03q^2 + 1$, find the equilibrium quantity and equilibrium price, the consumer surplus, and the producer surplus. Draw a simple plot and label these quantities on that plot.

16. *(10 points)* You are studying a new language. Already you know 2000 words and you estimate that you can learn new words at the rate of 1000 per year. Unfortunately you forget words at the rate of 10% per year. Solve the differential equation that models $W(t)$, the number of words you know at time t. How many words will you know in 5 years?

17. *(10 points)* Find the equation of the plane passing through the points $(1, 0, -1), (0, 1, 3), (1, -1, -2)$.

18. *(10 points)* The volume V, pressure P, and temperature T of a gas are related by the formula $V = \frac{kT}{P}$, where k is a constant. Compute $\frac{\partial V}{\partial T}$ and $\frac{\partial V}{\partial P}$. What is the approximate change in volume if the pressure increases from 30 to 32 and the temperature drops from 300 to 295?

19. *(10 points)* Use partial derivatives of first and second order to find all local minima, maxima, and saddle points of $g(x) = 3x^2 - 3xy + 3y^2 + 9y - 4$.