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| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
| Score | | | | | | | | | | |

UW Calculus 211
Final
Dec 20 , 2007

YOU MAY WANT TO USE THE FOLLOWING FORMULAS

$$\cos^2 x + \sin^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y.$$

SHOW ALL YOUR WORK.

CALCULATORS ARE NOT ALLOWED AND ARE NOT NEEDED.

NINE PROBLEMS.

GOOD LUCK,

M.KALAFAT

1. (5 pts) Compute the following value

$$\cos \frac{5\pi}{6}.$$

Hint : You may use the formulas given in the first page.

2. (5 pts) Find the derivative of the function $f(x) = e^{\cos(x^3)}$.

3. Compute the following integrals.

(a)(5 pts) $\int \sin t \cos^3 t dt$

(b)(5 pts) $\int \sin^3 x dx$

4. (10 pts) We would like to make an open-topped cardboard box with a square base. We are provided with 27 square inches of cardboard. What is the maximum possible volume of the box we can construct ?

5. Consider the function $f(x, y) = 6 + xy - \frac{1}{2}x^2 - \frac{1}{3}y^3$.

(a)(4 pts) Find its critical points.

(b)(6 pts) Determine the nature of each critical point(max/min/saddle etc).

6. Consider the function $f(x, y) = e^{x^2 - y^3}$.

(a)(6 pts) Compute the following partial derivatives

$$\frac{\partial f}{\partial x}(1, 1) \quad , \quad \frac{\partial f}{\partial y}(1, 1).$$

(b)(4 pts) Write down the equation of the tangent plane to the graph of f at the point $P_0 = (1, 1)$. Simplify your answer.

7. (10 pts) Find the equation of the least-squares line for the following points

$(0, 2)$, $(1, 2)$, $(2, 4)$.

8. Consider the points on the line $3x + y = 1$.

(a)(6 pts) If we are trying to minimize the distance between this line and the origin, find the function to minimize and write down the set of equations to solve.

(b)(4 pts) Find the point on the line which is closest to the origin.

9. (a)(3 pts) Find the solutions of the equation

$$3x^2 - 5x + 2 = 0.$$

(b)(3 pts) Find the natural domain of the function

$$f(x) = \frac{\sqrt{3x^2 - 5x + 2}}{2x + 7}.$$

(c)(4 pts) Compute the following limit

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 5x + 2}}{2x + 7}$$