

Math 211
Lecture 2

Spring 05/06
G.Meyer

Final

Show all work. No work = no credit, even if you have a correct answer.
References and calculator are not allowed.

1. (8 points)

Determine if the improper integral converges, and, if so, compute it:

$$\int_1^2 \frac{x}{\sqrt{x^2 - 1}} dx$$

2. (6 points)

Determine the intrinsic growth rate, the damping factor and the carrying capacity K of the logistic equation given by

$$\frac{dp}{dt} = 0.04p - 0.001p^2.$$

3. a) (5 points)

Find an equation of the plane through $(2,0,0)$, $(0,3,0)$ and $(0,0,12)$.

b) (5 points)

Find the linear approximation of $f(x, y) = x + y^2 + 5xy$ at $(2, 1)$.

4. (5 points)

Sketch the level curve of height 4 of $f(x, y) = x^2 - y + 3$.

5. (8 points)

Suppose the profit function of a company selling x units of one product and y units of another product is

$$P(x, y) = 12x + 9y - 450 - 0.01(4x^2 + xy + y^2).$$

Find the pair (x, y) that maximizes the company's profit and justify that it is the maximum by applying the second derivative test.

6. (10 points)

Find the maximum value of $f(x, y) = 6x - 8y$, subject to the constraint $3x^2 + 4y^2 = 7$.

7. (5 points)

Find the least squares line for this data: $(0, 1), (1, 2), (2, 4)$.

8. (5 points)

The value of a certain new computer changes at a rate of $10(t - 20)$ dollars per month, $0 \leq t \leq 36$. Compute the total change in the value of the computer over the first year.

9. (5 points)

Determine whether the function $f(x) = \frac{x-1}{x+1}$ has a global maximum/minimum on $[0, \infty)$. If so, where is it and what is its value?

10. Extra Credit(8 points)

Sketch the solution $y(t)$ of the initial value problem

$$\frac{dy}{dt} = y(4 - y), y(0) = 1.$$

Explain the shape of the graph: increasing/decreasing, concavity, inflection value and limiting value. Then do the same work if $y(0) = 5$.