

**Math 211**  
**Final Exam**  
**Lecture 1**

**Spring 2003**  
**S. Bolotin**

**Your Name:** \_\_\_\_\_

**Your TA:** \_\_\_\_\_

<b>PROBLEM</b>	<b>POINTS</b>	<b>SCORE</b>
I	25	
II	25	
III	15	
IV	15	
V	15	
VI	25	
VII	20	
VIII	15	
IX	20	
X	25	
<b>TOTAL</b>	<b>200</b>	

I. Find the derivatives of the following functions.

(a)  $f(x) = \tan(x^2 + 2x + 1)$ .

(b)  $f(x) = e^{2 \sin x}$ .

(c)  $f(x) = \frac{\tan x}{\cos x}$ .

(d)  $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

II. Find the integrals

(a)  $\int x^2 \cos(x^3) dx$

(b)  $\int_0^{\infty} x e^{-x^2} dx.$

(c)  $\int_0^{\pi/6} \frac{\sin x}{\cos^2(x)} dx.$

(d)  $\int x \cos(2x) dx.$

III. Find the average value of the function  $f(x) = x \sin x$  on the interval  $[0, \pi]$ .

IV. The graphs of  $y = \sin x$  and  $y = \cos x$  intersect at  $x = \pi/4$  and  $x = 5\pi/4$  but not in between. Find the area between the graphs for  $\pi/4 \leq x \leq 5\pi/4$ .

V. Let  $f(x, y) = \cos(x/y)$ .

(a) Find  $f_x, f_y$ .

(b) Find  $f_{yy}, f_{xy}$ .

VI. Let

$$f(x, y) = x^3 - 6x^2 + 9x + 2y^3 - 3y^2 - 12y + 1.$$

Find all critical points of  $f$  and identify them as a relative minimum, relative maximum, or neither.

VII. Let  $f(x) = 2x^3 - 12x^2 - 30x + 7$ .

(a) Find for what values of  $x$  is  $f$  increasing (decreasing).

(b) Find for what values of  $x$  is  $f$  concave up (down).

(c) Sketch the graph of  $f$ . You need only a qualitative picture: don't compute the values of the function.

VIII. Assume that the demand  $x$  for the product is related to its price  $p$  by the equation  $x^2 + 5xp + p^2 = 1500$ . Find the rate of change,  $\frac{dx}{dp}$ , in the demand with respect to the price for  $x = 20$  and  $p = 10$ .

IX. Ten years ago you deposited \$10000 at a constant interest rate compounded continuously. At present you have \$15000 in the account.

(a) Find how much money you will have in the account 20 years from now.

(b) Find when you will have \$30000. You don't need to simplify your answer.

- X. A dealer gets a car from the manufacturer for \$8000 and SUV for \$16000. He estimates that to sell  $x$  cars and  $y$  SUVs each month he will have to set the prices (in thousand dollars) at  $p = 20 + y - 2x$  for cars and  $q = 40 - 2y + x$  for SUVs. Find how many cars and SUVs he should sell each month to maximize his profits.