

Math 211  
Final Exam  
Lecture 1

Fall 2004  
S. Bolotin

Your Name: \_\_\_\_\_

Your TA: \_\_\_\_\_

PROBLEM	POINTS	SCORE
I	20	
II	20	
III	20	
IV	20	
V	20	
VI	20	
VII	20	
VIII	20	
IX	20	
X	20	
TOTAL	200	

Show all your work: no work - no credit. Leave your answers in exact forms (using  $e$ ,  $\ln 2$ ,  $\sqrt{2}$  and similar numbers). Circle your answer. Hand in your exam, together with the formula sheet, to your TA.

I. Find the derivatives of the following functions.

(a)  $f(x) = \frac{e^{2x}}{1+x^2}$

(c)  $f(x) = \ln(\tan x)$

(d)  $f(x) = \frac{1 - \sin x}{1 + \cos x}$

II. Find the integrals

(a)  $\int \frac{(\ln x)^3}{x} dx$

(b)  $\int_0^{\infty} x e^{-x} dx$

(c)  $\int_0^{\pi/2} \sin^2 x dx$

III. Suppose the area under the curve  $y = x^3$  between  $x = 0$  and  $x = 3$  is divided into 2 pieces by the line  $x = c$ . Find  $c$  such that the area of the left hand piece is twice the area of the right hand piece.

IV. Find the solution of the differential equation  $y' = \sqrt{y} \cos^2 x$  with the initial condition  $y(0) = 1$ .

V. Let  $f(x, y) = \sin(xy + 2y)$ .

(a) Find  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ .

(b) Find  $\frac{\partial^2 f}{\partial x \partial y}$ .

(c) Find the tangent plane to the graph  $z = \sin(xy + 2y)$  at the point  $P = (0, 0, 0)$ .

VI. Let

$$f(x, y) = x^3 - 6x^2 + 9x + 2y^3 - 3y^2 - 12y + 1$$

Find all critical points of  $f$  and identify them as a relative minimum, relative maximum, or neither.

VII. Let  $f(x) = 2x^3 - 9x^2 - 60x$ .

(a) Find intervals on which  $f$  increasing (decreasing).

(b) Find intervals on which  $f$  is concave up (down).

(c) Graph the function.

(d) Find the global maximal and minimum values of this function on the interval  $[-3, 2]$ .

VIII. The demand  $x$  (in thousands) for a certain product and the price  $p$  (in dollars) are related by the demand equation  $15x^2 - xp + 2xp^3 = 260$ . When the price is \$5, the demand is one thousand. Will the revenue  $R = xp$  increase or decrease if the price is raised a little?

- IX. A coffee shop buys premium coffee for \$2 per pound and regular brand for \$1 per pound. If the shop sells premium coffee for  $x$  dollars per pound, and regular for  $y$  dollars per pound, the shop will be able to sell  $40 - 5x + 2y$  pounds of premium coffee and  $30 - 5y + x$  pounds of regular coffee per week.
- (a) Find the weekly revenue function  $R(x, y)$ .

(b) Assuming that the only cost is the price paid for the coffee, find the weekly cost function  $C(x, y)$ .

(c) Determine the choice of  $x, y$  which maximizes the weekly profit  $P(x, y)$  and find the maximal possible profit.

- X. Find the maximal and minimal values of  $f(x, y, z) = x + 2y - z$  subject to the constrain  $x^2 + y^2 + z^2 = 24$ .