Math 211 **Spring 2004**

Final Exam **S. Bolotin**

Lecture 1

Your Name: ______________________________

Your TA: ______________________________

PROBLEM	POINTS	SCORE
I	20	
II	15	
III	20	
IV	20	
V	25	
VI	15	
VII	20	
VIII	25	
IX	20	
X	20	
TOTAL	200	

Show all your work: no work - no credit. Leave your answers in exact forms (using e, $\ln 2$, $\sqrt{2}$ and similar numbers). Circle your answer. Hand in your exam to your TA.

I. (20 points) Find the derivatives of the following functions.

(a) $f(x) = \tan(x^2 + 2x + 1)$.

(b) $f(x) = e^{2\sin x}$.

(c) $f(x) = \frac{\tan x}{\cos x}$.

II. (20 points) Find the integrals

(a) $\int x^2(\cos(x^3)+1)\,dx$

(b) $\int_0^{\pi/6} \frac{\sin x}{\cos^2(x)}\,dx$.

III. (20 points) The graphs of $y = x \sin x$ and $y = x$ intersect at $x = 0$ and $x = \pi/2$ but not in between. Find the area between the graphs for $0 \leq x \leq \pi/2$.

IV. (20 points) Let $f(x, y) = \cos(x/y)$.

(a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$.

(b) Find $\frac{\partial^2 f}{\partial x \partial y}$.

V. (25 points) Let

$$f(x, y) = x^3 - 6x^2 + 9x + 2y^3 - 3y^2 - 12y + 5.$$

Find all critical points of f and identify them as a local minimum, local maximum, or neither.

VI. (15 points) Assume that the demand q for the product is related to its price p by the equation $q^2 + 5qp + p^2 = 1500$. Find the rate of change, $\frac{dq}{dp}$, in the demand with respect to the price for $q = 20$ and $p = 10$.

VII. (20 points) Suppose ten years ago you deposited \$10000 in an account which pays interest rate 5% compounded continuously. For the next 20 years you keep depositing money continuously at the rate of \$1000 a year. Find how much money you will have 10 years from now.

VIII. (25 points) A dealer gets a car from the manufacturer for \$8000 and SUV for \$16000. He estimates that to sell x cars and y SUVs each month he will have to set the prices (in thousand dollars) at $p = 20 + y - 2x$ for cars and $q = 40 - 2y + x$ for SUVs. Find how many cars and SUVs he should sell each month to maximize his profits.

IX. (20 points) Find the maximal and minimal values of $f(x, y) = 3xy$ subject to the constraint $x^2 + 4y^2 = 16$.

X. (20 points) Find the solution of the differential equation $\frac{dy}{dt} = e^y \sin t$ with the initial condition $y(0) = 0$.