

Math 211

Exam 2

Dr. Chandarana

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Name:

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Discussion Section Time

Note: For full credit show all work and proper reasoning clearly. There will be very little or no credit for answers given without showing work. Incorrect reasoning may count against you. You are expected to give *exact* answers. No calculators are allowed on this exam. *After you finish writing the exam hand it to your TA along with the "cheat sheet".*

There are seven (7) pages (including this page) on this exam and an extra sheet for scratch work. Do not miss any pages by mistake.

Good Luck!

<i>problem</i>	<i>points</i>	<i>your score</i>
1	18	
2	16	
3	17	
4	18	
5	16	
6	15	
<i>total</i>	100	

1. (a) (6 points) Let $y = \sqrt{1 - x^2} + 3$. Find $\frac{dy}{dx}$ and show that

$$\frac{dy}{dx} = \frac{x}{3 - y}, \quad -1 < x < 1.$$

(b) (12 points) Solve the initial value problem

$$\frac{dy}{dx} = x^{\frac{3}{4}} + xe^{-x} \quad x \geq 0, \quad y(0) = 1.$$

2. (a) (10 points) If $y^3 + ye^{-x} + e^{2x} = 3$, find $\frac{dy}{dx}$.

(b) (6 points) The water level in a cylindrical tank with base radius of 4 feet is rising at the rate of 4 inches per hour. How fast is the volume of water increasing (in cubic feet per hour)?

3. (17 points) You bake and sell cakes daily in your bakery. The selling price is p (in dollars) per cake, and the number x of cakes that you sell is related to the selling price p by $x = 50 - 2p$. It costs you \$5 to produce each cake. Assuming that you sell all the cakes you bake everyday, what price should you charge per cake, and how many cakes should you produce per day to maximize profit? What is the maximum profit?

4. (a) (9 points) Find $I = \int \frac{1}{x^2 + x - 2} dx$.

(b) (9 points) Find $J = \int \frac{y \ln(1 + y^2)}{1 + y^2} dy$.

5. (16 points) Sketch the graph of a function $f(x)$ with the following properties:

- (i) The graph of $f(x)$ has a vertical asymptote at $x = 1$.
- (ii) $f'(x) > 0$ and $f''(x) > 0$ for $-\infty < x < 1$.
- (iii) $f'(x) > 0$ for $1 < x < 2$, $f'(2) = 0$ and $f(2) = 3$.
- (iv) $f''(x) < 0$ for $1 < x < 3$ and $f''(x) > 0$ for $x > 3$.
- (v) $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 1$.

6. (a) (15 points) Find all points where the function $f(x) = x^2[\ln(x)]^2$ has its local and global minimum and maximum values on the interval $(0, \infty)$ (if any) and find these extreme values. You may assume that $x^2[\ln(x)]^2 \rightarrow 0$ as $x \rightarrow 0+$. *DO NOT sketch the graph of $y = f(x)$.*

Scratch Work