

Math 211
Final Exam
Lecture 2

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Fall 2005
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Your Name: _____

Your TA: _____

PART	POINTS	SCORE
I	40	
II	40	
III	40	
IV	40	
V	40	
TOTAL	200	

Show all your work: no work - no credit. Leave your answers in exact forms (using e , $\ln 2$, $\sqrt{2}$ and similar numbers). Circle your answer. Hand in your exam to your TA.

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I. Functions.

(a)(5 points) Find $f(x) = |x - 3|e^{-\ln(x+2)}$ at point $x_0 = 2$.

(b)(10 points) Find a linear function which crosses the points $(0, 2)$ and $(2, 3)$.

(c)(10 points) Draw the graph of the function $f(x) = x^2 - 3x - 4$. Find vertex and x -intercepts. Do not use derivatives!

(d)(15 points) Find vertical, horizontal and slant asymptotes for the graph

$$y = \frac{(x^2 - 4x + 4)(x + 2)}{x^2 - 3x + 2}.$$

II. Limits. Find limits.

(a)(5 points) $\lim_{x \rightarrow \infty} \frac{x^3 - x + 1}{x + 2x^2 - 3x^3}$

(b)(10 points) $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 + 2x - 3}$

b . . .

(c)(10 points) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x(x+1)} \right)$

(d)(15 points) $\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}$

III. Derivatives.

(a)(5 points) Find derivative of $f(x) = e^{x^2+x^{-1}}$.

(b)(15 points) The equation $x^2 + y^2 = e^{xy}$ defines y as a function of x . Find the tangent line to the curve $y = f(x)$ at the point $(0, 1)$.

(c)(5 points) Find derivative of $f(x) = \frac{x^2 - \sqrt{x}}{\ln x}$.

(d)(15 points) Suppose $f(x) = x^3 - 6x^2 + 9x$.

(i) Find the critical points of $f(x)$.

(ii) Find where $f(x)$ has a local minimum or maximum.

(iii) What are the intervals on which $f(x)$ is concave up? Concave down?

IV. Integrals. Find integrals.

(a)(10 points) $\int x \ln x \, dx$

(b)(10 points) $\int x^2 (x^3 + 2)^{1/3} \, dx$

(c)(10 points) $\int_0^{\infty} x e^{-2x} dx$

(d)(10 points) $\int_0^2 x (x^2 + 5)^{1/2} dx$

V. Functions of two variables.

(a)(10 points) Suppose that $f(x, y) = (x + y^{-1})e^{x+2y}$. Find partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

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(b)(15 points) Find all critical points of the function $f(x, y) = y^3 - 3yx + 3x^2$.
Which points are local minimum or maximum?

(c)(15 points) Find the least-squares line for the following data: $(0, 1), (1, 2), (2, 4)$.