

Calculus and Introduction to Differential Equations
Math 213 Exam Two
Instructor: Jun Chen

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA session:

TA NAME	Tuesday	Thursday
Hua, Zheng	8:50am	8:50am
Van Essen, Anton	12:05pm	12:05pm
Van Essen, Anton	14:25pm	14:25pm

Problem/pt	P1/15	P2/20	P3/20	P4/20	P5/25	Total
Score						

1. (15 pts) Evaluate the improper integral $\int_1^{\infty} \frac{(\ln x)^2}{x^2} dx$.

2. (20 pts) Identify all the relative extrema and saddle points of $f(x, y) = x^2 + 4y^3 - 6xy - 1$.

3. (20 pts) Find the longest and shortest distances from the curve $x^2 + xy + y^2 = 1$ to the origin by the method of Lagrange multipliers.

4. (20 pt) Suppose that the price $P(x, y)$ of a product is a function of the cost x and the market demand y . Right now, $x = 100, y = 100$ and the price $P(100, 100) = 20$ dollars. If there is no change in market demand and the cost is lowered by 2% ($x = 98$), the price will be 16 dollars. On the other hand, if the cost is fixed and the market demand increases by 5% ($y = 105$), the price will be 25 dollars.
- (a) Use the above information to estimate $P_x(100, 100)$ and $P_y(100, 100)$.
- (b) Use the results in (a) to approximate the price when the cost is lowered by 5% and the market demand is reduced by 2% (Hint: Use total differential).

5. Evaluate the following double integrals:

(a) (12 pts) Let the region R be a triangle with three vertices $(0, 0)$, $(1, 0)$, $(1, 1)$.
Please evaluate

$$\iint_R x e^y dx dy.$$

(b) (13 pts) Evaluate

$$\int_0^1 \int_{e^x}^e \frac{x}{\ln y} dy dx.$$