Calculus and Introduction to Differential Equations
Math 213 Final Exam
Instructor: Jun Chen

No calculator allowed.
Write detailed work to obtain full credits.

Your Name:

Please circle your TA session:

TA NAME	Tuesday	Thursday
Hua, Zheng	8:50am	8:50am
Van Essen, Anton	12:05pm	12:05pm
Van Essen, Anton	14:25pm	14:25pm

Problem	P1	P2	P3	P4	P5	P6	P7	P8	Total
Score									

1. (15 pt) Solve differential equation: $\frac{dy}{dx} = xe^x$ with initial condition $y(0) = 3$.

2. (20 pts) Find the general solution for $x\frac{dy}{dx} + 2\,y - \frac{1}{x^2} = 0$, where $x > 0$.

3. (15 pts) Suppose in a geometric series $a_1 + a_2 + a_3 + \cdots$, we know $a_2 = 3, a_3 = 2$. Compute the sum of the series (Simplify the answer as a number or a fraction).

4. (10 pt) Suppose the annual interest rate is 8%, and the interest is compounded monthly. To set up an annuity of 5000 dollars in 5 years, how much is the monthly payment? (Do not simplify the answer)

5. (20 pts) Use L'Hospital's Rule to find the limit

$$\lim_{x \to 0} \frac{e^{x^3} - 1}{2\,x^2}.$$

6. (20 pts) Let $f(x) = \sqrt{x^2+1}$. Use $P_2(x)$ (the Taylor polynomial of degree 2 for $f(x)$) to estimate $\int_0^1 f(x)dx$. (Simplify your answer)

7. (25 pts) Find the Taylor series for $f(x) = e^{-x^2} - \frac{1}{2x+1}$ (write the first 3 terms). Also find the interval of convergence.

8. (25 pts) (Mixing Problem) Suppose a tank contains 20 gallons of salt water with concentration 1 ounce/gal. If we pour pure water into the tank at the rate of 1 gallon per minute, and the mixture flows out at the rate of 2 gallons per minute, what is the concentration of the salt water in the tank after 10 minutes?