MATH 213: MIDTERM2 (FALL 2006)

Name:______________________________

Section: _________ TA:______________

Score:

Problem 1.______________________

Problem 2.______________________

Problem 3.______________________

Problem 4.______________________

Problem 5.______________________

Problem 6.______________________

Total:______________________

Instruction: Show all work. No work = no credit, even if you have a correct answer. References and calculator are not allowed.

Problem 1 (15 points): Consider the function $z = f(x, y) = x^3 e^{\frac{y}{x}}$.

(a) (10 points) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. Then write down the total differential dz.

(b) (5 points) Use the total differential to find the approximate value of $(0.99)^3 e^{\frac{0.01}{0.99}}$

Problem 2 (10 points): Let $f(x, y) = 2x^3 + y^2 - 6xy$. Find all its critical points. Determine if they are points for relative maximum, relative minimum or saddle points.

Problem 3 (15 points): An ant is walking on a circular wire on the xy-plane satisfying the equation $x^2 + y^2 = 1$. Suppose the temperature at (x, y) on the xy-plane is given by $f(x, y) = x^2 + xy + y^2$ hundred ^oF. Then where are the coolest places *on the wire* that the ant can stay?

Problem 4 (10 points): Evaluate the double integral

$$\int\int_R e^{y^3}\,dxdy,$$

where R is the region in the xy-plane bounded by $x = 0$, $y = 1$ and $y = \sqrt{x}$.
(Hint: choose a good order of integration.)

Problem 5 (10 points): Solve the initial value problem:

$$\frac{dy}{dx} + \frac{1}{x}y = x^2; \quad y(2) = 3.$$

Problem 6 (10 points): Consider the initial value problem

$$\frac{dy}{dx} = 1 + xy; \quad y(0) = 0.$$

Use Euler's method with stepsize $h = 0.1$ to find the approximate values of the solution at $x = 0.1, 0.2$ and 0.3.