Math 213 Third In-class Exam

Version A

Please show your computations, not only your answers.

Room B239, 9:55am - 10:45am, April 15, 2005
Márton Balázs

NAME:
Dis. Session:

Problem	Points	Score
1	20	
2	25	
3	15	
Total:	60	

1. Let $r(t)$ denote the distance of the laser head of a CD-player from the center of the disk at time t. Each time the disk makes a complete revolution, the head advances by a small fixed distance. As the local speed of the disk is constant, the time one revolution takes is proportional to the inverse of the circumference, that is, the inverse of r. Therefore, the speed of the head is proportional to the inverse of r. In fact, r (measured in millimeters) satisfies the following differential equation:

$$\frac{dr}{dt} = \frac{19}{r}$$

where t is measured in minutes. We also know that playing starts at time zero at $r(0) = 23$ (millimeters).

(a) (10 points) Solve the initial value problem for $r(t)$.

(b) (5 points) A full CD is 74 minutes long. Find the radius of the disk by computing $r(74)$.

(c) (5 points) Find $r(37)$, the distance of the head from the center at half playing time.

2. The amount $A(t)$ (in thousand dollars) on my bank account earns continuously compounded interest at 5% annual rate. To cover my living expenses, I charge my account by a continuous money flow having rate $15 + t$ (in thousand dollars/year), t years from now. Therefore $A(t)$ satisfies

$$\frac{dA}{dt} = 0.05A - (15 + t).$$

Solve the above differential equation, and find the value of $A(60)$ (the amount on my account 60 years from now)

(a) (15 points) with an initial amount of $A(0) = 600$ (thousand dollars).

(b) (5 points) with an initial amount of $A(0) = 700$ (thousand dollars).

(c) (5 points) with an initial amount of $A(0) = 800$ (thousand dollars).

3. Compute $y(1)$ (and show each step of your computation!) if $y(x)$ satisfies

$$\frac{dy}{dx} = -2y, \qquad y(0) = 1,$$

(a) (10 points) using Euler's method, with step size $h = 0.25$,

(b) (5 points) by solving the initial value problem.