Math 213, Final Exam, Dec 20, 2002
Instructor: Shi Jin

NAME:

Please show details of your work

1. (50 points). Find each integral

$$(a)\ \int \frac{x^3}{e^{3x^4}}\,dx\,, \qquad (b)\ \int_1^3 \frac{\sqrt{\ln x}}{x}\,dx\,, \qquad (c)\ \int_1^e x^3 \ln x\,dx\,,$$

$$(d)\ \int \frac{2^{x+1}-5^{x-1}}{10^x}\,dx\,, \qquad (e)\ \int_2^\infty \frac{dx}{x^2+x-2}\,.$$

2. (30 points) Find the limits (n is a positive integer)

$$(a)\quad \lim_{x\to 0} \frac{(\sqrt{1+x^2}+x)^n-(\sqrt{1+x^2}-x)^n}{x},$$

$$(b)\quad \lim_{x\to 1} \frac{x^{n+1}-(n+1)x+n}{(x-1)^2}.$$

3. (30 points)

(a) Find the following integral

$$\int\int \sqrt{x+y}\,dx\,dy$$

over the triangle bounded by $x = 0, y = 0$ and $x + y = 1$.

(b) Change the order of integration:

$$\int_{-6}^{2}\int_{x^2/4-1}^{2-x} f(x,y)\,dy\,dx\,.$$

4. (20 points) Find the Taylor series for the given function and give the interval of convergence of the series:

$$x^2 \ln(x+3)\,.$$

5. (30 points) Find the extrema of

$$z = Ax^2 + 2Bxy + Cy^2,$$

where A, B, C are constants, under the condition

$$x^2 + y^2 = 1\,.$$

6 (40 points) Are the following infinite series converge? If they do find the infinite sum.

(a) $\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{3^2}\right)+\cdots+\left(\frac{1}{2^n}+\frac{1}{3^n}\right)+\cdots$

(b) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}\cdots+\frac{1}{n\cdot(n+1)}+\cdots$