K. Zhu

MATH 213: FINAL (FALL 2005)

Name:__________________________________

Section __________ TA:________________

Score:

Problem 1._________________________

Problem 2._________________________

Problem 3._________________________

Problem 4._________________________

Problem 5._________________________

Problem 6._________________________

Problem 7._________________________

Total:_________________________

Instruction: Show all work. No work = no credit, even if you have a correct answer. References and calculator are not allowed.

Problem 1 (25 points): Find each integral:
(a) (5 points) $\int(x^{\frac{1}{4}} + x^{-2} + x)dx$

(b) (5 points) $\int(-\frac{4}{x} + 3^x)dx$

(c) (5 points) $\int_1^2 x^2 \ln x dx$

(d) (10 points) $\int_0^{+\infty} \frac{2x+1}{(x^2+x+5)^2} dx$

Problem 2 (10 points): Let $z = f(x, y) = e^{x+y^2}$.

(a) (5 points) Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) (5 points) Write down dz. Then find the approximate value of $f(0.02, 0.01)$.

Problem 3 (10 points): Suppose that the total cost to produce x square mirrors and y round mirrors is given by

$$C(x, y) = x^2 + 2y^2 + xy + 10.$$

If a total of 12 mirrors must be made, how should production be allocated such that the total cost is minimized?

Problem 4 (15 points): Evaluate the following double integrals:

(a) (5 points) $\int\int_R xydxdy$, where R is the region in the xy-plane bounded by $y = 2x$, $y = 3x$ and $x = 1$.

(b) (10 points) $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dxdy$. (Hint: change the order of integration.)

Problem 5: (a)(5 points) Find the Taylor polynomial of degree 2 for $f(x) = (1+x)^{\frac{1}{5}}$ at $x = 0$.

(b) (10 points) Find the Taylor series for $f(x) = \frac{x^2}{1-x^3}$ at $x = 0$. What's its interval of convergence?

Problem 6 (10 points): Consider the geometric series

$$1 - \frac{1}{3} + \frac{1}{9} + ... + (-\frac{1}{3})^{n-1} + ...$$

(a) (5 points) Find the sum of the first 4 terms.

(b) (5 points) Find the sum of the whole infinite series.

Problem 7 (15 points): Using L'Hospital's Rule to find the following limits:
(a) (5 points) $\lim_{x\to 2}\frac{x^4-16}{x-2}$

(b) (10 points) $\lim_{x\to 0}\frac{e^{x^2}-1}{x\ln(1+x)}$