Math 217 – FINAL EXAM – Spring 2005

YOUR NAME: ______________________________

Circle your TA's name and the meeting time of your Discussion Session:

Seth Case	11:00am	2:25pm
Stephen Hruska	12:05pm	3:30pm
James Hunter	7:45am	8:50am
Ekin Ozman	8:50am	9:55am
Gabriel Pretel	9:55am	1:20pm
Patrick Rault	12:25pm	

Show all your work. Answers without showing all work receive zero points.

1. *(10 points)* Identify the critical points and find the minimum and maximum value of $f(t) = \sin t + \cos t$ on the interval $[0, \pi]$. Then, identify the inflection points and sketch its graph.

2. *(20 points)* Compute the following:

(a) $\displaystyle\int \frac{5y}{\sqrt{2y^2-3}}\,dy$

(b) $\displaystyle\int_{-1}^{1} \sqrt{4-x^2}\,dx$

(c) $\displaystyle\int \frac{\cosh\sqrt{y}}{\sqrt{y}}\,dy$

(d) What is the average value of $f(x)=\sqrt{5-x}$ on the interval $[0,5]$?

3. *(20 points)* At 10:00 A.M. one car was 150 miles due west from a second car. If the first car drove east at 50 miles per hour and the second car drove southwest at 75 miles per hour, when were they closest together?

4. *(20 points)* Compute the following derivatives:

(a) $\frac{d}{dt}\left(\sqrt{e^{t^2}} + e^{\sqrt{t^2}}\right)$

(b) $\frac{d}{dx}\int_{-1}^{x^3} \cos^2 t \, dt$

(c) $\frac{d}{dx}\left[\frac{\sqrt{x+5}(2x+1)}{(x-1)\sqrt[3]{3x-4}}\right]$

(d) If $2^{xy} = 2 + x\tan^{-1} y$, then $\frac{dy}{dx} =?$

5. *(10 points)*

(a) Compute the sum $\sum_{i=100}^{200} \left[\frac{1}{i^2} - \frac{1}{(i+1)^2} \right]$

(b) Compute the sum $\sum_{k=3}^{50} (k-1)(k-2)$

6. *(10 points)* Compute $\int_{-2}^{1}(2x^2-3)dx$ using the **definition** of the definite integral.

7. *(15 points)* *(a)* Find the length of the curve given parametrically by $x = e^t \sin t,\ y = e^t \cos t,\ 0 \le t \le 2\pi$.

(b) Find the general solution of the differential equation $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$; then find the particular solution that satisfies the condition $y = 4$ at $x = 1$.

8. *(20 points)* If the brakes of a car, when fully applied, produce a constant deceleration of 9 feet per second per second, what is the shortest distance in which the car can be braked to a halt from a speed of 60 miles per hour? *Hint: set up a differential equation by using the fact that "deceleration" means negative acceleration; also, use the fact that 60 miles per hour is the same as 88 feet per second.*

9. *(25 points) (a)* A bacterial population grows at a rate proportional to its size. Initially, it is 20000, and after 5 days it is 70000. What is the population after 9 days?

(b) A radioactive substance has a half-life of 1700 years. If there were 12 grams initially, how much would be left after 400 years?

(c) How long does it take for money to double at 6% interest compounded monthly?

10. *(15 points)*

(a) (5 points) Sketch the region bounded by $y = \sinh x$, $y = 0$, and $x = \ln 2$, and calculate the area of this region.

(b) (10 points) Sketch the region between the graphs of $x = -6y^2 + 4y$ and $x + 3y - 2 = 0$, and calculate the area of this region.

11. *(20 points)* Compute the following limits:

(a) $\lim_{x \to \frac{\pi}{2}} \frac{ln(\sin x)^2}{x - \frac{\pi}{2}}$

(b) $\lim_{x \to 0} \frac{x - \tan^{-1} x}{2x^3}$

(c) $\lim_{\theta \to \frac{\pi}{2}} (\sec\theta - \tan\theta)$

(d) $\lim_{x \to 0+} (\sin x)^{5/x}$

12. *(15 points) (a)* Write the expression $\frac{4+\sqrt{-81}}{7-\sqrt{-64}}$, in the form $a + bi$, where a and b are real numbers.

(b) Find all real and complex solutions of the equation $x^3 + 125 = 0$.

(c) Express the complex number $2\sqrt{3} + 2i$ in trigonometric form; then, represent it in a system of coordinates where the horizontal axis represents the real part and the vertical axis represents the imaginary part.