MATH 217 MIDTERM - APRIL 24, 2003

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1. Find (a) $\frac{d}{dx}((ln\ x^2)^x)$,

(b) $\int e^{x-e^x}\ dx$.

2. (a) What explicitly is the Integrating Factor for the first order linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x).$$

(b) Solve

$$sin\ x\ \frac{dy}{dx} + 2y\ cos\ x = sin\ 2x,$$

if $y = 2$ when $x = \pi/6$.

3. Find (a) $\frac{d}{dx}(tanh^{-1}(cos\ x))$ (make sure to simplify your answer),

(b) $\int_0^{\pi/4} \frac{sin\ \theta}{1+cos^2\theta}\ d\theta$.

4. Evaluate $\int_0^1 \frac{x-1}{\sqrt{x^2+1}}\, dx$.

5. Find (a) $\frac{d}{dx}(cosh^4\ x)$,

(b) $\int sin^3\ 3t\ \sqrt{cos\ 3t}\ dt.$

Acceleration due to gravity is 32 feet per second squared.

The area of a circle of radius r is πr^2; its circumference is $2\pi r$.

The area of a triangle with base b and height h is $\frac{1}{2}bh$.

The volume of a cylinder of height h and radius r is $\pi r^2 h$.

The volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$; its surface area is $4\pi r^2$.

Pythagoras's theorem: $a^2 + b^2 = c^2$, where a, b, c are the sides of a right triangle with c the hypotenuse.

The circle with center the origin and radius r has equation $x^2 + y^2 = r^2$.

$tan(t) = \frac{sin(t)}{cos(t)}, cot(t) = \frac{cos(t)}{sin(t)}$,

$sec(t) = \frac{1}{cos(t)}, csc(t) = \frac{1}{sin(t)}$.

$sin(2t) = 2sin(t)cos(t), cos(2t) = cos^2 t - sin^2 t, cos^2 t + sin^2 t = 1, sec^2 t = 1 + tan^2 t$,

$sin(\frac{t}{2}) = \pm\sqrt{\frac{1-cos(t)}{2}}, cos(\frac{t}{2}) = \pm\sqrt{\frac{1+cos(t)}{2}}$.

$sin(0) = 0 = cos(\frac{\pi}{2}), cos(0) = 1 = sin(\frac{\pi}{2}), sin(\frac{\pi}{6}) = \frac{1}{2} = cos(\frac{\pi}{3})$,

$sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} = cos(\frac{\pi}{4}), sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} = cos(\frac{\pi}{6})$.

If $a \neq 0$, then the roots of $ax^2 + bx + c = 0$ are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$a^m a^n = a^{m+n}, \frac{a^m}{a^n} = a^{m-n}, (a^m)^n = a^{mn}, a^{-n} = 1/a^n$.

$\frac{d}{dx}(sin(x)) = cos(x), \frac{d}{dx}(cos(x)) = -sin(x), \frac{d}{dx}(tan(x)) = sec^2(x)$,
$\frac{d}{dx}(cot(x)) = -csc^2(x), \frac{d}{dx}(sec(x)) = sec(x)tan(x), \frac{d}{dx}(csc(x)) = -csc(x)cot(x)$.
$sinh(x) = (e^x - e^{-x})/2, cosh(x) = (e^x + e^{-x})/2$.
$\frac{d}{dx}(sinh(x)) = cosh(x), \frac{d}{dx}(cosh(x)) = sinh(x), \frac{d}{dx}(tanh(x)) = sech^2(x)$,
$\frac{d}{dx}(coth(x)) = -csch^2(x), \frac{d}{dx}(sech(x)) = -sech(x)tanh(x), \frac{d}{dx}(csch(x)) = -csch(x)coth(x)$.
$\frac{d}{dx}(ln(x)) = 1/x, \frac{d}{dx}(e^x) = e^x$.
$\frac{d}{dx}(sin^{-1}(x)) = 1/\sqrt{1-x^2}, \frac{d}{dx}(cos^{-1}(x)) = -1/\sqrt{1-x^2}$,
$\frac{d}{dx}(tan^{-1}(x)) = 1/(1+x^2), \frac{d}{dx}(sec^{-1}(x)) = 1/(|x|\sqrt{x^2-1})$.
$\frac{d}{dx}(sinh^{-1}(x)) = 1/\sqrt{x^2+1}, \frac{d}{dx}(cosh^{-1}(x)) = 1/\sqrt{x^2-1}$,
$\frac{d}{dx}(tanh^{-1}(x)) = 1/(1-x^2), \frac{d}{dx}(sech^{-1}(x)) = -1/(x\sqrt{1-x^2})$.
$\int a^u du = a^u/(ln(a)) + C, \int 1/u du = ln|u| + C$.
$\int sin(u)du = -cos(u) + C, \int cos(u)du = sin(u) + C, \int sec^2(u)du = tan(u) + C$,
$\int csc^2(u)du = -cot(u) + C, \int sec(u)tan(u)du = sec(u) + C, \int csc(u)cot(u)du = -csc(u) + C$,

$\int tan(u)du = -ln|cos(u)| + C, \int cot(u)du = ln|sin(u)| + C, \int sec(u)du = ln|sec(u) + tan(u)| + C, \int csc(u)du = ln|csc(u) - cot(u)| + C$.
$\int 1/\sqrt{a^2-u^2}du = sin^{-1}(u/a) + C, \int 1/(a^2+u^2)du = (1/a)tan^{-1}(u/a) + C$,
$\int 1/(a^2-u^2)du = (1/2a)ln|(u+a)/(u-a)| + C$,
$\int 1/(u\sqrt{u^2-a^2})du = (1/a)sec^{-1}(|u/a|) + C$.