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Exam 1

Name: _____

Problem	Score
1	
2	
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8	
Total	

Please circle your TA's name:

Nick Addington

Rikki Bostelmann

Hongnian Huang

Karl Mahlburg

Rob Owen

Seyfi Turkelli

Li Wang

Weidong Yin

Absolutely no calculators, notes, or books are allowed. You must show all your work, and explain your reasoning to receive credit for your answers.

On the last page of the exam are formulas that may or may not come in handy on some problems.

Be sure to check your answers whenever possible.

Good luck!

1. [5 points each] For each of the following, write the sum in sigma notation and then find the value.

(a) $2 + 4 + 6 + \cdots + 30$

(b) $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \frac{3}{64}$

2. [7 points each] Find the following indefinite integrals.

(a) $\int \sqrt{t} \, dt$

(b) $\int \left(x^2 - \frac{1}{x^3} \right) dx$

2. *continued*

(c) $\int (x^2 - 2 \cos x) dx$

(d) $\int \sec x \tan x dx$

3. [8 points each] Evaluate the following definite integrals.

$$(a) \int_{-\frac{1}{2}}^0 (2x + 1)^{\frac{1}{3}} dx$$

$$(b) \int_0^{\sqrt[3]{2}} 3x^2 \sin^3(x^3 - 1) dx$$

4. [6 points] Evaluate the following limit by writing it as a definite integral and then evaluating that integral. Make sure you show some justification for the definite integral you're using.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}$$

5. [10 points] Let \mathcal{R} be the region bounded by $y = x^2 + 1$ and $y = x + 3$. Sketch \mathcal{R} and find its area.

6. [10 points] Let \mathcal{R} be the region bounded by $y = 1 + \cos x$ and the x -axis between $x = -\pi$ and $x = \pi$. Draw a sketch of \mathcal{R} , and then set up—but do not evaluate—the integral representing the volume of the solid generated by revolving \mathcal{R} about the x -axis.

7. [10 points] Find the length of the curve given by $x = 2t^3$, $y = t^2$, $0 \leq t \leq 1$.

8. [10 points] Suppose your car is capable of decelerating at a rate of 22 ft/s^2 when your brakes are fully applied. You're driving along a dark country road late one night when a young deer casually strolls out into the road. The deer stops, turns, and stares right into your headlights. If you are exactly 200 ft from little Bambi and going 60 mph (88 ft/s) when you slam on your breaks, will you be able to stop without hitting him?

Some Formulas

Special Sum Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

geometric sum:

$$\sum_{k=0}^n ar^k = a + ar + ar^2 + \cdots + ar^n = \frac{a-ar^{n+1}}{1-r} \quad (r \neq 1)$$

Double-Angle & Half-Angle Formulas for Sine and Cosine

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 1 - 2 \sin^2 u$$

$$= 2 \cos^2 u - 1$$

$$\sin^2 \frac{v}{2} = \frac{1 - \cos v}{2}$$

$$\cos^2 \frac{v}{2} = \frac{1 + \cos v}{2}$$

Geometric Formulas

Circle: area = πr^2 , circumference = $2\pi r$

Triangle: area = $\frac{1}{2}bh$

Trapezoid: area = $\frac{1}{2}(b_1 + b_2)h$