

Math 217
Spring 2005

C. Gómez
April 4, 2005

Exam 2

Name: _____

Problem	Score
1	
2	
3	
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7	
8	
Total	

Please circle your TA's name:

Nick Addington

Rikki Bostelmann

Hongnian Huang

Karl Mahlburg

Rob Owen

Seyfi Turkelli

Li Wang

Weidong Yin

Absolutely no calculators, notes, or books are allowed. You must show all your work, and explain your reasoning to receive credit for your answers.

On the last page of the exam are formulas that may or may not come in handy on some problems.

Be sure to check your answers whenever possible.

Good luck!

1. [3 points each] Give the definition of each of the following functions.

(a) $g(x) = \ln x$

(b) $y = \sinh x$

(c) $f(x) = \log_5 x$

2. [6 points each] Solve the following equations for x .

(a) $e^x + 2 = 8e^{-x}$

(b) $\log_2 x - \log_2(x + 1) = \log_2\left(\frac{1}{8}\right)$

(c) $\frac{1}{2} \sin^{-1}(x - 3) = \frac{\pi}{4}$

3. Consider the function $f(x) = \tanh x = \frac{\sinh x}{\cosh x}$.

(a) [5 points] What type of indeterminate form does $\lim_{x \rightarrow \infty} f(x)$ have? Explain your answer using the definitions of $\sinh x$ and $\cosh x$.

(b) [5 points] Explain why L'Hôpital's Rule **does not** help in evaluating $\lim_{x \rightarrow \infty} f(x)$.

(You do not have to find the actual limit.)

4. [6 points each] Find the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$

(b) $\lim_{x \rightarrow 0} (e^x + 3x)^{\frac{1}{x}}$

5. [5 points each] Differentiate the following functions.

(a) $f(x) = \sinh x^3 + 2^x + x^\pi$

(b) $g(x) = \ln \sin^2\left(\frac{x}{2}\right)$

(c) $y = \sin^{-1}(2x^2)$

6. [7 points each] Evaluate the following integrals.

(a) $\int \frac{1}{x^2 - 2x + 10} dx$

(b) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x} dx$

(c) $\int \tanh x \ln(\cosh x) dx$

7. [6 points] Solve the initial value problem

$$y' - \frac{y}{x} = 3x^3; \quad y = 3 \text{ when } x = 1$$

8. [9 points] A population of bacteria is known to grow exponentially. If 4 million are observed initially and 9 million after 2 days, how many will be present after 3 days?

Your Cheat Sheet

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), \quad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}, \quad -1 < x < 1$$

$$\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right), \quad 0 < x \leq 1$$

Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}, \quad x \geq 1$$

$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1 - x^2}, \quad -1 < x < 1$$

$$\frac{d}{dx} \operatorname{sech}^{-1} x = \frac{-1}{x\sqrt{1 - x^2}}, \quad 0 < x \leq 1$$

Double-Angle & Half-Angle Formulas for Sine and Cosine

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u & \cos 2u &= \cos^2 u - \sin^2 u \\ & & &= 1 - 2 \sin^2 u \\ & & &= 2 \cos^2 u - 1 \end{aligned}$$

$$\begin{aligned} \sin^2 \frac{v}{2} &= \frac{1 - \cos v}{2} \\ \cos^2 \frac{v}{2} &= \frac{1 + \cos v}{2} \end{aligned}$$

Other

half-life of carbon 14: 5730 years